MATH 230 - Lecture 1 (01/11/2011)

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Course web page: www.wsu.edu/~kbala/Math230.html
(see Syllabus and other details here).

Linear Algebra - Motivating example

Butch M. Cougar has 5 hrs (7pm-midnight), and $48 dollars to spare.
"Math"

Two options:
- get tutoring → $8/hr
- party → $16/hr

How many hrs to get tutored, and how many to party?

Let x_1 = # hrs of tutoring
x_2 = # hrs of partying } variables

Model:

$$\left.\begin{aligned} x_1 + x_2 &= 5 \quad \text{(total hrs)} \\ 8x_1 + 16x_2 &= 48 \quad \text{(total money)} \end{aligned}\right\} \text{System of two linear equations}$$

A generic linear equation : $a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$

coefficients

right-hand side (rhs)

In general, we talk about a system of m-linear equations in n variables.

The graph of a linear equation is a straight line.

Here are some equations that are NOT linear:

$$2x_1x_2 + 3x_3 = -4$$

$$-\sqrt{x_2} + 3x_3^5 = 0$$

non-linear terms.

The coefficients (a_i) and rhs (b) can be real or complex numbers.
In Math 230, we will deal with only real numbers.

A **solution** is a set of values for $x_1, x_2, \ldots$ for which each equation in the system is true (or holds).
The **solution set** is the set of all solutions.

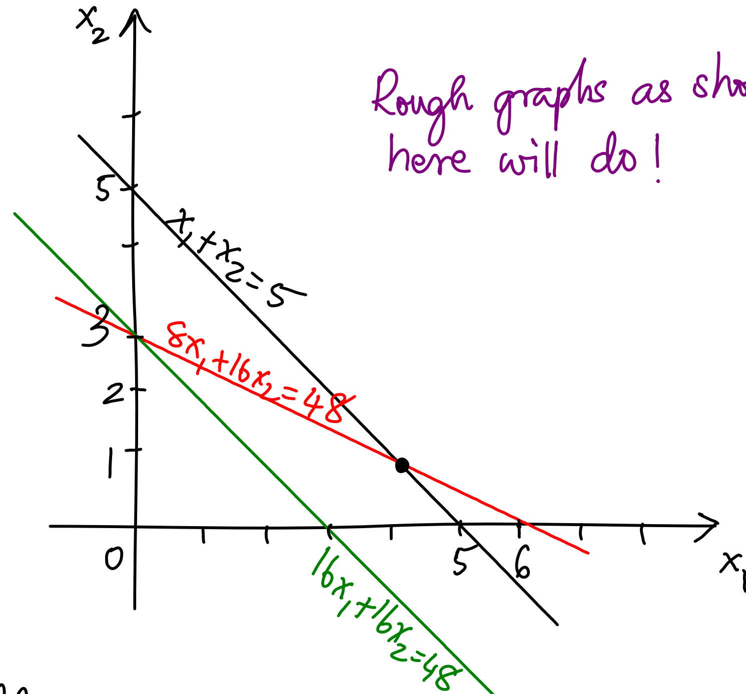
$$\left.\begin{aligned} x_1 + x_2 &= 5 \quad \text{—(1)} \\ 8x_1 + 16x_2 &= 48 \quad \text{—(2)} \end{aligned}\right\} \text{A solution is given by} \quad \left.\begin{aligned} x_1 &= 4 \\ x_2 &= 1 \end{aligned}\right\}$$

find solution by plotting the graphs of the two equations.

Find two points per line:

$x_1+x_2=5$ } $(5,0)$ and $(0,5)$ are points on this line

$8x_1+16x_2=48$ } $(6,0)$ & $(0,3)$ are points



Rough graphs as shown here will do!

How many solutions?

The 2 lines intersect at exactly one point. So, the solution set is $\{(4,1)\}$, i.e., it is a singleton set.

There are two other extreme cases

* The 2 lines do not intersect $\Rightarrow$ "implies" system has no solution, OR the system is **inconsistent**.

e.g., fees for tutoring is \$16/hr now. The system is

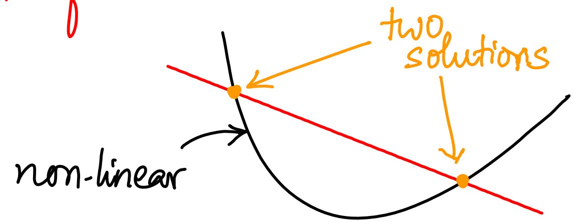
$$\left.\begin{aligned} x_1+x_2&=5 \\ 16x_1+16x_2&=48 \end{aligned}\right\}$$ The two lines are parallel.

* The 2 lines intersect at <u>ALL</u> pts $\Rightarrow$ the system has infinitely many solutions.

e.g., tutoring is \$16/hr, and Butch has only 3 hrs to spare:

$$\left.\begin{aligned} x_1+x_2&=3 \\ 16x_1+16x_2&=48 \end{aligned}\right\}$$ The two lines are the same!

Note: We never get, say, 2 solutions with lines!
We could get 2 points of intersection if we had curve(s) instead of lines.



In summary, a system of linear equations can have

1. no solution; $\longrightarrow$ inconsistent system
2. exactly one solution; OR
3. infinitely many solutions } consistent system

The idea of plotting lines will not work in higher dimensions.
In general, we use elimination to solve a system of linear equations.

$\hookrightarrow$ From the original system, obtain an "equivalent" system that is "simpler".

both systems have the same solution set.

$$\underbrace{\left.\begin{aligned} x_1 + x_2 &= 5 \quad \text{—(1)} \\ 8x_1 + 16x_2 &= 48 \quad \text{—(2)} \end{aligned}\right\}}_{\text{original system}} \longrightarrow \left.\begin{aligned} x_1 + 0x_2 &= 4 \\ 0x_1 + x_2 &= 1 \end{aligned}\right\} \text{ simpler equivalent system.}$$

How? Eliminate x_1 from equations (2), (3), ...,
eliminate x_2 from equations (1), (3), ..., etc.

$$\begin{aligned} x_1 + x_2 &= 5 \quad \text{---}(1) \\ 8x_1 + 16x_2 &= 48 \quad \text{---}(2) \end{aligned} \xrightarrow{-8\times(1)+(2)} \begin{aligned} x_1 + x_2 &= 5 \quad \text{---}(1') \\ 8x_2 &= 8 \quad \text{---}(2') \end{aligned} \quad (2')\times\tfrac{1}{8}$$

$$\begin{aligned} x_1 + x_2 &= 5 \quad \text{---}(1'') \\ x_2 &= 1 \quad \text{---}(2'') \end{aligned} \xrightarrow{(1'')-(2'')} \begin{aligned} x_1 &= 4 \\ x_2 &= 1 \end{aligned}$$

This procedure is called Gaussian elimination. The operations are Elementary row operations (**EROs**).

they do not change the solution set.

There are **three** types of EROs.

1. (**Replacement**): Replace an equation with the sum of itself and a multiple of another equation.
2. (**Interchange**): Swap two equations (or rows). just change the order in which equations are written
3. (**Scaling**): Multiply an equation (row) by a **non-zero** number.
   If you multiply by zero, you remove that equation!

More compact representation: Avoid writing $x_1, x_2, \ldots$ each time.

$$\begin{aligned} x_1 + x_2 &= 5 \\ 8x_1 + 16x_2 &= 48 \end{aligned} \qquad \begin{bmatrix} 1 & 1 \\ 8 & 16 \end{bmatrix} \text{ is the matrix of coefficients.}$$

A **matrix** is a rectangular array of numbers. It has rows and columns.

The <u>size</u> of a matrix: (# rows) $\times$ (# columns)

said as "by"

$\begin{bmatrix} 1 & 1 \\ 8 & 16 \end{bmatrix}$ is a 2×2 matrix.

We include the rhs numbers along with the matrix of coefficients:

$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 8 & 16 & 48 \end{array}\right] \longrightarrow$ the **augmented matrix** of the system of linear equations

just a separator line; separates the rhs from coefficients.

$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 8 & 16 & 48 \end{array}\right]$ is a 2×3 matrix.

We can do EROs directly on the augmented matrix.

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 8 & 16 & 48 \end{array}\right] \xrightarrow{R_2-8R_1} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 8 & 8 \end{array}\right] \xrightarrow{R_2\times(1/8)} \left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 1 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 1 \end{array}\right]$$

compact notation for EROs.