MATH230 – Lecture 25 (04/12/2011)

Computer project: Illustration

Say we have $A = \begin{bmatrix} 0 & 5 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 2 \end{bmatrix}$.

$[m,n] = \text{size}(A)$ gives $m=3$, $n=4$.

$n_e = 0$ the # EROs (is zero at start)

Iteration 1 of main while loop

$i=1, j=1$ start at top-left corner

$$\begin{array}{rc} & \quad j \\ \begin{array}{r} i_1 = i \rightarrow \\ i_1 = 2 \rightarrow \\ i_1 = 3 \rightarrow \end{array} & \begin{bmatrix} 0 & 5 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 2 \end{bmatrix} \end{array}$$

$i_1 = 1$, $A(1,1) = 0 \Rightarrow$ go to next row: $i_1 \rightarrow 2$ $(1+1)$

$i_1 = 2$ $A(2,1) = 0 \Rightarrow$ go to next row: $i_1 \rightarrow 3$ $(2+1)$

$i_1 = 3$ $A(3,1) = 2 \neq 0$ pivot found!

$i_{NZ} = i_1 = 3$; nzfound $=1$;

Store the index of the pivot row

so, stop the "while nzfound==0 ..." loop when you try to repeat it next time

In pseudocode, == is checking for equality, while = is used for assignment.

$i = i+1$ → take current value of i, add 1, and store the result in i itself.

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if j==5
    DoSomething;
end
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→ DoSomething is run if j is 5.

The same syntax is used in MATLAB — == for comparison and = for assignment.

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nzfound = 1, $i_{nz} = 3$, $i=1$, $j=1$ now.

$i_{nz} \neq i$ (pivot row is below current row), so swap $R_{i_{nz}}$ and R_i ($R_3 \rightleftharpoons R_1$ here)

If the pivot were located in Row i itself, we would not have to do a row swap here. The goal is to get the pivot located in Position (i,j) itself, assuming there exists a pivot.

$$A = \begin{bmatrix} 0 & 5 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 2 & 3 & 0 & 2 \end{bmatrix} \xrightarrow{R_1 \rightleftharpoons R_3} \begin{array}{c} \\ i\rightarrow \\ \\ \\ \end{array} \begin{array}{c} \begin{array}{cccc} j & & & \end{array} \\ \begin{bmatrix} 2 & 3 & 0 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 5 & 2 & 4 \end{bmatrix} \end{array}$$

$n_e = 0+1 = 1$

increment #EROs by 1.

$$R_i \times \frac{1}{A(i,j)} = R_1 \times \frac{1}{A(1,1)} = R_1 \times \frac{1}{2}$$

Scale the pivot so that it becomes 1.

$$A = \begin{bmatrix} 1 & 3/2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 5 & 2 & 4 \end{bmatrix}$$

$n_e = 1+1 = 2;$

Here, $A(2,1)=0$, $A(3,1)=0$ already. So we do not any replacement EROs to make this pivot column $(j=1)$ a unit vector.

$i=i+1 \Rightarrow i=1+1=2.$ → go to next row

$j=j+1 \Rightarrow j=1+1=2$ → go to next column.

END of Iteration 1 here.

Iteration 2

$$\begin{array}{cc} & \begin{array}{cccc} & j & & \end{array} \\ \begin{array}{c} \\ i\rightarrow \\ \\ \end{array} & \begin{bmatrix} 1 & 3/2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 5 & 2 & 4 \end{bmatrix} \end{array} \quad \text{(} i_1 \rightarrow 0,\ i_1 \rightarrow 5 \text{)} \quad (i_1=3)$$

We repeat the same set of calculations on the smaller → 2×3 matrix.

$j=2$ now

Start with $i_1=i=2$, nzfound $=0$. → Look for a pivot in Column $j=2$, in Row $i=2$ or lower.

$A(2,2)=0$. So, $i_1=i_1+1=3$. → go to next row.

$A(3,2)=5\neq 0$. nzfound $=1$; $i_{nz}=i_1=3$ → save pivot row

Now we have nzfound $==1$, $i_{nz}=3$, $i=2$.

Swap R_2 and R_3 $(R_{i_{nz}}$ and $R_i)$ $R_2 \rightleftarrows R_3$ → to get pivot in $A(i,j)$.

$$\begin{array}{cc} & \begin{array}{cccc} & j & & \end{array} \\ \begin{array}{c} \\ i\rightarrow \\ \\ \end{array} & \begin{bmatrix} 1 & 3/2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 5 & 2 & 4 \end{bmatrix} \end{array} \xrightarrow{R_2 \rightleftarrows R_3} \begin{bmatrix} 1 & 3/2 & 0 & 1 \\ 0 & 5 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad n_e = 2+1 = 3$$

$$R_i \times \frac{1}{A(i,j)} = R_2 \times \frac{1}{5} \longrightarrow \begin{bmatrix} 1 & 3/2 & 0 & 1 \\ 0 & 1 & 2/5 & 4/5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad n_e = 3+1 = 4$$

Scale to make pivot = 1.

$i=2$ now, $(j=2)$. → zero out non-pivot entries in column j

for $i_1 = 1,2,3$ but $i_1 \neq i$, i.e., for $i_1 = 1, 3$, zero out $A(i_1, j)$.

$i_1 = 1 \Rightarrow A(i_1, j) = A(1, j) = \frac{3}{2}$. So, do $R_1 - \frac{3}{2} R_2$.

In general, do $R_{i_1} - A(i_1, j) \times R_i$ → current pivot row

$$\begin{bmatrix} 1 & 3/2 & 0 & 1 \\ 0 & 1 & 2/5 & 4/5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 - \frac{3}{2}R_2} \begin{array}{c c} & \begin{array}{cccc} & j & & \end{array} \\ & \\ i \rightarrow & \end{array} \begin{bmatrix} 1 & 0 & -3/5 & -1/5 \\ 0 & 1 & 2/5 & 4/5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad n_e = 4+1 = 5$$

for $i_1 = 3$, $A(i_1, j) = A(3,2) = 0$, so no replacement ERO is needed.

$i = i+1 = 3$ → go to next row

$j = j+1 = 3$. → go to next column

END of Iteration 2

Iteration 3

$$\begin{array}{c c} & \begin{array}{cccc} \quad & \quad & j & \quad \end{array} \\ & \\ & \\ i \rightarrow & \end{array} \begin{bmatrix} 1 & 0 & -3/5 & -1/5 \\ 0 & 1 & 2/5 & 4/5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

→ Notice that we already have the RREF of A. But the function runs through all rows and all columns before it finishes.

start with $i_1 = i = 3$, nzfound $= 0$.

$A(i_1, j) = A(3,3) = 0$. nzfound $= 0$ still. → nothing more done.

END of Iteration 3.

Iteration 4

$i = 3$, → since nzfound $= 0$, i is not incremented here.

$j = j+1 = 4$ → go to next column.

$$\begin{array}{c c} & \begin{array}{cccc} \quad & \quad & \quad & j \end{array} \\ & \\ & \\ i & \end{array} \begin{bmatrix} 1 & 0 & -3/5 & -1/5 \\ 0 & 1 & 2/5 & 4/5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

→ nothing more done.

END of RREF!