MATH 230 - Lecture 7 (02/01/2011)

Homogeneous and non-homogeneous systems

$$x_1 + 2x_2 + 3x_3 = 0$$
$$2x_1 + 5x_2 + 4x_3 = 0$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \end{bmatrix} \xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \end{bmatrix}$$

$$\xrightarrow{R_1 - 2R_2} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & -2 \end{bmatrix} \left\} \begin{array}{l} x_1 = -7x_3 \\ x_2 = 2x_3 \end{array} \right.$$

x_3 free

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -7 \\ 2 \\ 1 \end{bmatrix} s, \quad s \in \mathbb{R}$$

$$x_1 + 2x_2 + 3x_3 = 2$$
$$2x_1 + 5x_2 + 4x_3 = -3$$

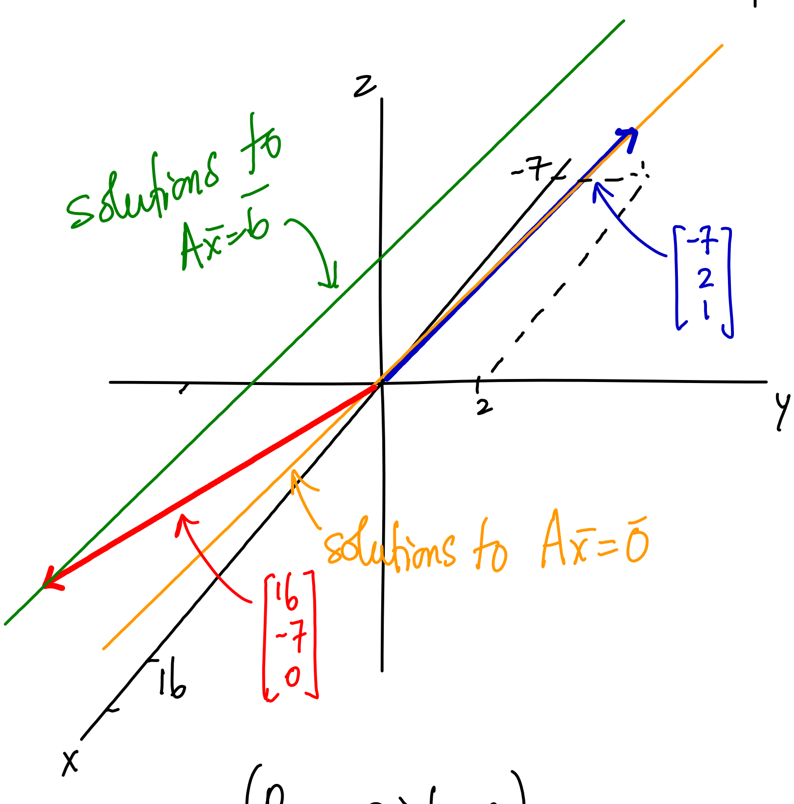
$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 2 & 5 & 4 & -3 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 2 \\ 0 & 1 & -2 & -7 \end{array}\right]$$

$$\xrightarrow{R_1 - 2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 7 & 16 \\ 0 & 1 & -2 & -7 \end{array}\right]$$

$x_1 = 16 - 7x_3$
$x_2 = -7 + 2x_3$
x_3 free

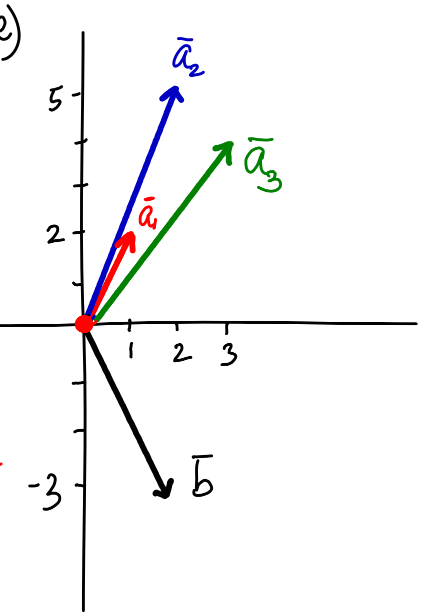
$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 16 \\ -7 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 2 \\ 1 \end{bmatrix} s, \quad s \in \mathbb{R}$$

$$\underset{\bar{a}_1}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}} x_1 + \underset{\bar{a}_2}{\begin{bmatrix} 2 \\ 5 \end{bmatrix}} x_2 + \underset{\bar{a}_3}{\begin{bmatrix} 3 \\ 4 \end{bmatrix}} x_3 = \underset{\bar{b}}{\begin{bmatrix} 2 \\ -3 \end{bmatrix}}$$



(Row picture)

(column picture)



To get to $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$, we first get to origin by taking a linear combination of $\bar{a}_1, \bar{a}_2, \bar{a}_3$, and then jump to $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$.

To get to $\bar{b}$ by taking linear combinations of $\bar{a}_1, \bar{a}_2, \bar{a}_3$, we can first get to $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (origin) by taking combination(s) of $\bar{a}_1, \bar{a}_2, \bar{a}_3$, and then just add $\bar{b}$.

Section 1.6 – Applications

Prob 4, pg 63

4. Suppose an economy has four sectors, Agriculture (A), Energy (E), Manufacturing (M), and Transportation (T). Sector A sells 10% of its output to E and 25% to M and retains the rest. Sector E sells 30% of its output to A, 35% to M, and 25% to T and retains the rest. Sector M sells 30% of its output to A, 15% to E, and 40% to T and retains the rest. Sector T sells 20% of its output to A, 10% to E, and 30% to M and retains the rest.

a. Construct the exchange table for this economy.

b. **[M]** Find a set of equilibrium prices for the economy.

The exchange table gives how production, or revenue, is exchanged between the various sectors in the economy. From the exchange table, we should be able to write down linear equations involving the "prices" or "production" from each sector.

Exchange table for economy

output	A	E	M	T	purchased by (or used by)
	0.65	0.30	0.3	0.2	A
	0.10	0.10	0.15	0.1	E
	0.25	0.35	0.15	0.3	M
	0	0.25	0.40	0.4	T

add up to 1. Gives the fractions of the total output from A being used by each sector.

Note that numbers in each column add up to 1.

Let p_A, p_E, p_M, p_T be the prices. For equilibrium, we must have "input" price = "output" price. From the exchange table, we get the following equations.

$$\overset{-0.35}{\cancel{0.65}}\,p_A + 0.3p_E + 0.3p_M + 0.2p_T = \cancel{p_A}\ 0$$

$$0.1\,p_A + \overset{-0.9}{\cancel{0.1}}\,p_E + 0.15p_M + 0.1p_T = \cancel{p_E}\ 0$$

$$0.25\,p_A + 0.35p_E + \overset{-0.85}{\cancel{0.15}}\,p_M + 0.3p_T = \cancel{p_M}\ 0$$

$$0\,p_A + 0.25p_E + 0.4p_M + \overset{-0.6}{\cancel{0.4}}\,p_M = \cancel{p_T}\ 0$$

This is the homogeneous system $A\bar{p} = \bar{0}$, where

$$A = \begin{bmatrix} -0.35 & 0.3 & 0.3 & 0.2 \\ 0.1 & -0.9 & 0.15 & 0.1 \\ 0.25 & 0.35 & -0.85 & 0.3 \\ 0 & 0.25 & 0.4 & -0.6 \end{bmatrix} \quad \text{and} \quad \bar{p} = \begin{bmatrix} p_A \\ p_E \\ p_M \\ p_T \end{bmatrix}.$$

Switched to MATLAB - see course web page for commands and output!

From MATLAB, we get the reduced echelon form of A as

$$RREF(A) = \begin{bmatrix} 1 & 0 & 0 & -2.028 \\ 0 & 1 & 0 & -0.531 \\ 0 & 0 & 1 & -1.168 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$