MATH 230 - Lecture 8 (02/03/2011)

Use words carefully!

~~system is unique~~ $\to$ solution to the system is unique.

Market equilibrium problem (Prob 4. pg 63)

p_A, p_E, p_M, p_T prices of sectors A, E, M, T. $A\bar{p} = \bar{0}$

Reduced echelon form of A is

$$\begin{bmatrix} 1 & 0 & 0 & -2.03 \\ 0 & 1 & 0 & -0.53 \\ 0 & 0 & 1 & -1.17 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

p_T is a free variable

2 decimal places will do, as p's are prices

$p_A = 2.03\, p_T$,
$p_E = 0.53\, p_T$,
$p_M = 1.17\, p_T$.

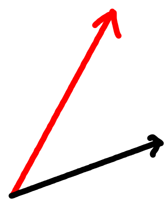
Hence, if $p_T = \$100$, then $p_A = \$203$, $p_E = \$53$, and $p_M = \$117$, are the equilibrium prices.

We could have chosen p_A, or p_E, or p_M as the free variable here! Equivalently, the equilibrium prices can be expressed in terms of any one of the four variables. For instance,

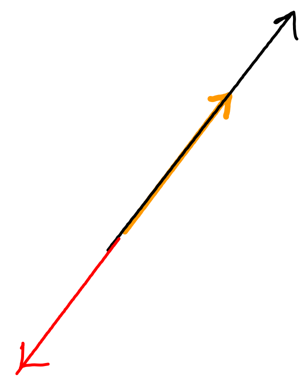
$p_T = \left(\frac{1}{2.03}\right) p_A$. Hence $p_E = \left(\frac{0.53}{2.03}\right) p_A$ and $p_M = \left(\frac{1.17}{2.03}\right) p_A$.

(See course web page for MATLAB session!)

Linear Independence (Section 1.7)



linearly independent vectors



linearly dependent vectors.

$\{\bar{v}_1, \ldots, \bar{v}_n\}$ are vectors in $\mathbb{R}^m$ (i.e., $\bar{v}_j \in \mathbb{R}^m$ for each j).

Def: The (set of) vectors $\{\bar{v}_1, \ldots, \bar{v}_n\}$ (is) are linearly independent (LI) if $x_1\bar{v}_1 + \cdots + x_n\bar{v}_n = \bar{0}$ has only the trivial solution. Else the vector are linearly dependent (LD).

Equivalently, the columns of a matrix A are LI if and only if $A\bar{x} = \bar{0}$ has only the trivial solution.

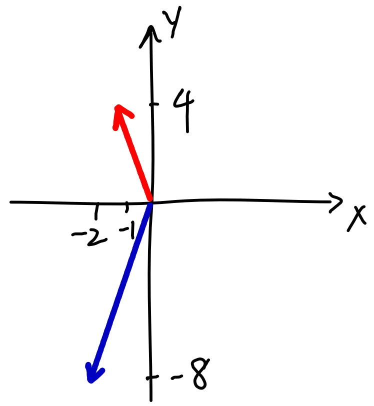
(This happens when there is a pivot in every column of A, i.e., there are no free variables).

Prob 4, Pg 71

$\begin{bmatrix}-1\\4\end{bmatrix}, \begin{bmatrix}-2\\-8\end{bmatrix}$ are the vectors LI? Justify.

$$A = \begin{bmatrix}-1 & -2\\4 & -8\end{bmatrix} \xrightarrow{R_2+4R_1} \begin{bmatrix}-1 & -2\\0 & -16\end{bmatrix}$$

No free variables. Hence $A\bar{x}=\bar{0}$ has only the trivial solution. So the vectors are LI.



→ The vectors are indeed LI!

Prob 12, Pg 71

$\begin{bmatrix}2\\-4\\1\end{bmatrix}, \begin{bmatrix}-6\\7\\-3\end{bmatrix}, \begin{bmatrix}8\\h\\4\end{bmatrix}$ For what h are these vectors LD?

$$A = \begin{bmatrix}2 & -6 & 8\\-4 & 7 & h\\1 & -3 & 4\end{bmatrix} \xrightarrow[R_2+4R_3]{R_1-2R_3} \begin{bmatrix}0 & 0 & 0\\0 & -5 & h+16\\1 & -3 & 4\end{bmatrix} \xrightarrow{R_1 \rightleftharpoons R_3} \begin{bmatrix}1 & -3 & 4\\0 & -5 & h+16\\0 & 0 & 0\end{bmatrix}$$

Every column does not have a pivot (or, x_3 is a free variable). Hence $A\bar{x}=\bar{0}$ has non-trivial solutions. So, the vectors are LD for $h \in \mathbb{R}$.

Special cases of LI/LD of $\{\bar{v}_1, \dots, \bar{v}_n\}$ in $\mathbb{R}^m$

① One vector : $\bar{v}_1 \in \mathbb{R}^m$ is LI, unless $\bar{v}_1 = \bar{0}$

$x_1 \bar{v}_1 = \bar{0}$ only when $x_1 = 0$. But if $\bar{v}_1 = \bar{0}$, then $x \in \mathbb{R}$. Hence $\{\bar{0}\}$ is LD.

② Two vectors $\bar{v}_1, \bar{v}_2$ are LI if they do not lie on the same line. Equivalently, $\bar{v}_1, \bar{v}_2$ are LD if $\bar{v}_1 = c\,\bar{v}_2$ for a scalar c.