Calculus III (Math 273, Section 2) – Fall 2014
Exam 1

- There are **eight** problems and **four** pages in this exam.
- Show all work, and provide appropriate **justifications** where required.
- Calculators, cell phones, laptops, or any other electronic devices are **not** allowed.
- Good luck!

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1. **(14)** Find the domain and range of the function $f(x)$ given below, and identify its level curves. Sketch one typical level curve. Is the domain open or closed? Is the domain bounded?

$$f(x, y) = \ln(x^2 + y^2 - 1).$$

2. **(12)** Find the first partial derivatives with respect to each variable of the functions in each case.

   (a) $f(x, y) = \dfrac{2+y}{x+\cos y}$.

   (b) $g(x, y, z) = \cos(2xy + 3y - 5z)$.

3. **(12)** Find all second order partial derivatives of the function $f(x, y) = x^2 - 3xy + \sin y + 5e^x$.

4. **(12)** Show that if $w = f(s)$ is a differential function of s and if $s = y + 5x$, then

$$\frac{\partial w}{\partial x} - 5\frac{\partial w}{\partial y} = 0.$$

5. **(12)** Find $\dfrac{dy}{dx}$ at $P(0, \ln 2)$ when y is defined implicitly as a function of x by $2xy - e^{x+y} - 2 = 0$.

6. **(14)** The derivative of the function $f(x, y, z)$ at point P is greatest in the direction of $2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$, and this derivative has value 7. What is the derivative of f at P in the direction $3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$?

7. **(12)** Is there a direction $\mathbf{u}$ in which the rate of change of $f(x, y) = 2x^2 - 5xy + 4y^2$ at the point $P(2, 1)$ equals 4? **Justify** your answer.

8. **(12)** Decide whether each of the following statements is *True* or *False*. **Justify** your answer.

   (a) If a set is unbounded, then it must also be open.

   (b) The partial derivatives f_x and f_y of a function $f(x, y)$ exist at all points in its domain.

   (c) The directional derivative of f in a direction $\mathbf{u}$ is a vector pointing in the same direction as $\mathbf{u}$.

   (d) If $|\nabla f| = 2$ at a point P_0, then there is a direction $\mathbf{u}$ along which the derivative of f at P_0 is 1.