MATH 273 – Lecture 11 (09/30/2014)

Estimating changes in a specific direction

In 1D, change in $f(x)$ at $x = p_0$ is estimated by

$$\underbrace{df}_{\text{differential of } f \text{ at } x=p_0} = \underbrace{f'(p_0)}\cdot dx \quad \text{for small increment } dx$$

→ derivative × increment

Extending to higher dimensions,

$$df = \left((\nabla f)_{P_0} \cdot \hat{u} \right) ds,$$ where ds is the change in the direction of $\hat{u}$.

directional derivative × increment in the direction of $\hat{u}$

Prob 21 By about how much will $g(x,y,z) = x + x\cos z - y\sin z + y$ change when $P(x,y,z)$ moves from $P_0(2,-1,0)$ toward the point $P_1(0,1,2)$ a distance of $ds = 0.2$ units?

$$\nabla g = \frac{\partial g}{\partial x}\hat{i} + \frac{\partial g}{\partial y}\hat{j} + \frac{\partial g}{\partial z}\hat{k}$$

$$= (1 + \cos z - 0 + 0)\hat{i} + (0 + 0 - \sin z + 1)\hat{j} + (0 - x\sin z - y\cos z + 0)\hat{k}$$

$$= (1 + \cos z)\hat{i} + (1 - \sin z)\hat{j} - (x\sin z + y\cos z)\hat{k}$$

$$(\nabla g)_{P_0} = (1 + \underbrace{\cos 0}_{1})\hat{i} + (1 - \underbrace{\sin 0}_{0})\hat{j} - (2\underbrace{\sin 0}_{0} + (-1)\underbrace{\cos 0}_{1})\hat{k}$$

$$= 2\hat{i} + \hat{j} + \hat{k}.$$

Direction $\bar{u} = \overrightarrow{P_0P_1} = (0-2)\hat{i} + (1-(-1))\hat{j} + (2-0)\hat{k}$

$P_0(2,-1,0)$ $\quad = -2\hat{i} + 2\hat{j} + 2\hat{k}$

$P_1(0,1,2)$ $\quad \|u\| = \sqrt{(-2)^2+(2)^2+(2)^2} = 2\sqrt{3}.$

So, $\quad \hat{u} = \dfrac{\bar{u}}{\|\bar{u}\|} = \dfrac{1}{2\sqrt{3}}\left(-2\hat{i}+2\hat{j}+2\hat{k}\right) = \dfrac{1}{\sqrt{3}}\left(-\hat{i}+\hat{j}+\hat{k}\right).$

$$\left(D_{\hat{u}}g\right)_{P_0} = \left(\nabla g\right)_{P_0}\cdot \hat{u} = \left(2\hat{i}+\hat{j}+\hat{k}\right)\cdot\frac{1}{\sqrt{3}}\left(-\hat{i}+\hat{j}+\hat{k}\right)$$

$$= \frac{1}{\sqrt{3}}\left(2\times -1 + 1\times 1 + 1\times 1\right) = 0.$$

So $\quad dg = \left(D_{\hat{u}}g\right)_{P_0}\cdot ds = 0.(0.2) = 0.$

11·3

Review for Exam 1

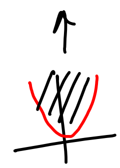
Exam 1 covers 13.1, 13.3, 13.4, and 13.5.

- 13.1 → domain, range, level curves, open/closed, bounded/unbounded
- 13.3 → partial derivatives
- 13.4 → chain rule, branch diagrams
- 13.5 → gradient & directional derivative

Practice Exam

⑧ True/False

(a) False. Take $y \geq x^2$ is closed, as it includes its boundary $y = x^2$. But it is unbounded



(b) False. We draw two branch diagrams, one for each independent variable.

(c) True. Follows from properties of ∇f.

(d) False. $(D_{\hat{u}} f) = \nabla f \cdot \hat{u} = |\nabla f| \underbrace{|\hat{u}|}_{=1} \cos\theta$

If $|\nabla f| = 0$, then $(D_{\hat{u}} f) = 0$ in all directions $\hat{u}$.

2(a). $f(x,y) = \dfrac{x+y}{xy-1}$

$$\frac{\partial f}{\partial x} = \frac{(xy-1)(1+0)-(x+y)(y-0)}{(xy-1)^2} = \frac{\cancel{xy}-1-\cancel{xy}-y^2}{(xy-1)^2} = -\frac{(y^2+1)}{(xy-1)^2}$$

$f(x,y)$ is symmetric w.r.t x and y, so

$$\frac{\partial f}{\partial y} = \frac{-(x^2+1)}{(yx-1)^2} = -\frac{(x^2+1)}{(xy-1)^2}.$$

$$f(y,x) = \frac{y+x}{yx-1} = \frac{x+y}{xy-1} = f(x,y)$$

Alternatively, we could evaluate $\frac{\partial f}{\partial y}$ directly:

$$\frac{\partial f}{\partial y} = \frac{(xy-1)(0+1)-(x+y)(x-0)}{(xy-1)^2} = \frac{(\cancel{xy}-1)-x^2-\cancel{xy}}{(xy-1)^2}$$

$$= \frac{-(x^2+1)}{(xy-1)^2}.$$