MATH 273 – Lecture 14 (10/09/2014)

Wind chill factor problem – continued...

31. Wind chill factor Wind chill, a measure of the apparent temperature felt on exposed skin, is a function of air temperature and wind speed. The precise formula, updated by the National Weather Service in 2001 and based on modern heat transfer theory, a human face model, and skin tissue resistance, is

$$W = W(v, T) = 35.74 + 0.6215\,T - 35.75\,v^{0.16} + 0.4275\,T \cdot v^{0.16},$$

where T is air temperature in °F and v is wind speed in mph. A partial wind chill chart is given.

v (mph) \ T(°F)	30	25	20	15	10	5	0	−5	−10
5	25	19	13	7	1	−5	−11	−16	−22
10	21	15	9	3	−4	−10	−16	−22	−28
15	19	13	6	0	−7	−13	−19	−26	−32
20	17	11	4	−2	−9	−15	−22	−29	−35
25	16	9	3	−4	−11	−17	−24	−31	−37
30	15	8	1	−5	−12	−19	−26	−33	−39
35	14	7	0	−7	−14	−21	−27	−34	−41

a. Use the table to find $W(20, 25)$, $W(30, -10)$, and $W(15, 15)$.

b. Use the formula to find $W(10, -40)$, $W(50, -40)$, and $W(60, 30)$.

c. Find the linearization $L(v, T)$ of the function $W(v, T)$ at the point $(25, 5)$.

d. Use $L(v, T)$ in part (c) to estimate the following wind chill values.

i) $W(24, 6)$ ii) $W(27, 2)$

iii) $W(5, -10)$ (Explain why this value is much different from the value found in the table.)

at $P_0(v_0, T_0) = (\underset{v_0}{25}, \underset{T_0}{5})$

$$L(v,T) = W(v_0, T_0) + \left.\frac{\partial W}{\partial v}\right|_{P_0}(v - v_0) + \left.\frac{\partial W}{\partial T}\right|_{P_0}(T - T_0)$$

$$\frac{\partial W}{\partial v} = 0 + 0 - (35.75)(0.16)v^{(0.16-1)} + 0.4275 \cdot T \cdot (0.16)\, v^{(0.16-1)}$$
$$= -(35.75)(0.16)v^{-0.84} + (0.4275)(0.16)\,T\,v^{-0.84}$$

$$\frac{\partial W}{\partial T} = 0 + 0.6215 - 0 + 0.4275 \cdot v^{0.16}$$
$$= 0.6215 + 0.4275 \cdot v^{0.16}$$

At $P_0(25, 5)$, we get $W(v_0, T_0) = -17.41$, $W_v|_{P_0} = -0.36$, and $W_T|_{P_0} = 1.34$. Hence

$$L(v,T)|_{P_0} = -17.41 - 0.36(v - \overset{v_0}{25}) + 1.34(T - \overset{T_0}{5})$$
$$= -15.09 - 0.36v + 1.34T.$$

(d) $W(24,6) \approx L(24,6) = -15.71$, which is very close to $W(24,6)$ itself!

approximately

$L(27,2) = -22.14$, while $W(27,2) = -22.143$.

But, $L(5,-10) = -30.26$, while $W(5,-10) = -22.26$!

The values are very different because $(5,-10)$ is not near $P_0(25,5)$. The linearization is valid (or accurate) only close to the point at which the linearization is taken.

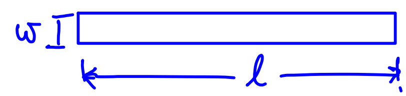
Here are the commands used in Octave (same as MATLAB):

```
octave:2> v0=25;T0=5;
octave:3> W = 35.74 + 0.6215*T - 35.75*v^(0.16) + 0.4275*T*v^(0.16);
error: `T' undefined near line 3 column 20
octave:3> v=v0; T=T0; W = 35.74 + 0.6215*T - 35.75*v^(0.16) + 0.4275*T*v^(0.16)
W = -17.409
octave:4> W_v = -(35.75)*(0.16)*v^(0.16-1) + 0.4275*0.16*T*v^(0.16-1)
W_v = -0.36004
octave:5> W_T = 0.6215 + 0.4275*v^(0.16)
W_T =  1.3370
octave:6> -17.409 - 0.36004*(-25) + 1.337*(-5)
ans = -15.093
octave:7> L = -15.093 - 0.36004*v + 1.337*T
L = -17.409

octave:8> v=24;T=6;
octave:9> L = -15.093 - 0.36004*v + 1.337*T
L = -15.712
octave:10>  W = 35.74 + 0.6215*T - 35.75*v^(0.16) + 0.4275*T*v^(0.16)
W = -15.710

octave:11> v=27;T=2;
octave:12> L = -15.093 - 0.36004*v + 1.337*T
L = -22.140
octave:13>  W = 35.74 + 0.6215*T - 35.75*v^(0.16) + 0.4275*T*v^(0.16)
W = -22.143

octave:14> v=5;T=-10;
octave:15> L = -15.093 - 0.36004*v + 1.337*T
L = -30.263
octave:16>  W = 35.74 + 0.6215*T - 35.75*v^(0.16) + 0.4275*T*v^(0.16)
W = -22.256

octave:17> v=5;T=20;
octave:18> L = -15.093 - 0.36004*v + 1.337*T
L =  9.8468
octave:19>  W = 35.74 + 0.6215*T - 35.75*v^(0.16) + 0.4275*T*v^(0.16)
W =  12.981
```

We now do a problem using the total differential.

Prob 51 We are calculating the area of a thin long rectangle by measuring length and width. Which dimension should we measure more carefully so as to minimize error in the area computed?

Area $A = lw$ $\qquad l$ = length w = width



l is much larger than w

The total differential $dA = A_l \, dl + A_w \, dw$.

But $A_l = w$ and $A_w = l$. So

$$dA = w\,dl + l\,dw$$

We can think of dA as the error in computing the area, and dl and dw as the errors in measuring length and width, respectively.

To keep dA small, we need to keep dw small, as the latter term is getting multiplied by l, which is large. Since dl is getting multiplied by w, which is smaller, dl does not affect dA as much as dw.

So, measure width, i.e., smaller dimension, more accurately.

Prob 50 (in Hw 6)

$$\frac{1}{R} = \frac{1}{\underset{x}{R_1}} + \frac{1}{\underset{y}{R_2}} \qquad (1) \qquad R(R_1, R_2) \quad R(x, y)$$

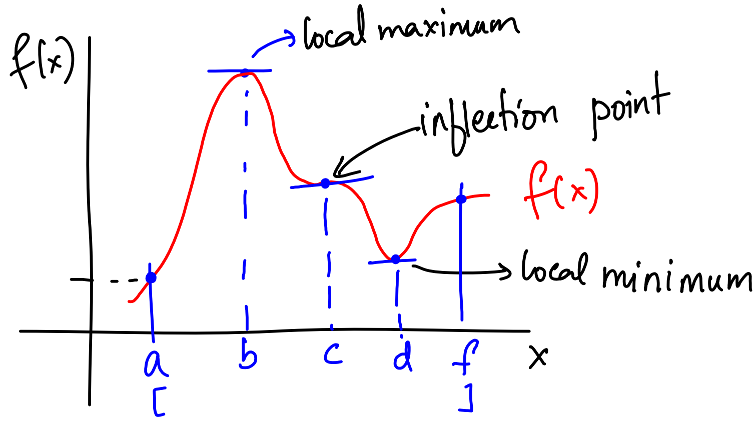
$$dR = \frac{\partial R}{\partial R_1} dR_1 + \frac{\partial R}{\partial R_2} dR_2$$

Do implicit differentiation of (1) w.r.t R_1 and then R_2, then solve for dR

Extreme Values and Saddle Points (Section 13.7)

In 1D: local maxima/minima and inflection points

→ all are **critical points**.



$f'(x) = 0$ at $x = b, c, d$

$f''(x) < 0$ at $x = b \Rightarrow$ local max

$f''(x) = 0$ at $x = c \Rightarrow$ inflection

$f''(x) > 0$ at $x = d \Rightarrow$ local min

To find all critical points, we also examine the boundary of the domain.

We extend these ideas to 2D and higher dimensions!

Def $f(a,b)$ is a local maximum (local minimum) value of f if $f(a,b) \geq f(x,y)$ $(f(a,b) \leq f(x,y))$ for all x,y in an open disk centered at (a,b).