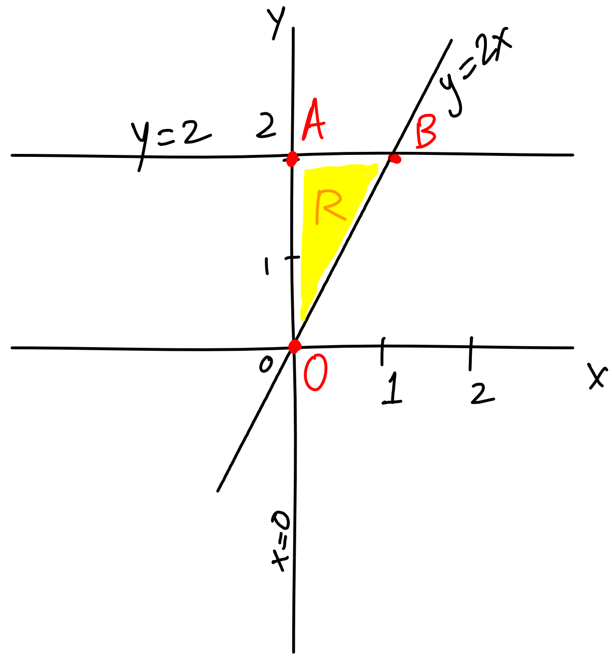
MATH 273 – Lecture 17 (10/21/2014)

Prob 31, continued..

$f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$

Region R : $\triangle OAB$



(i) No critical points interior to R. Only critical point (with $f_x = f_y = 0$) is $B(1,2)$, which is on the boundary.

(ii) Check the boundary segments.

on $\overline{OA}$:

$(x=0) \Rightarrow y^2 - 4y + 1 = 0$ giving $A(0,2)$ as the possible critical point, but A is also on the boundary, i.e., at the end of line segment $\overleftrightarrow{OA}$.

on $\overline{AB}$

$y=2$: $f(x,2) = 2x^2 - 4x + (2)^2 - 4(2) + 1 = 2x^2 - 4x - 3$.

$f'(x,2) = 4x - 4 = 0$ giving $x = 1$, and hence $B(1,2)$ is a critical point. But B is not in the middle of $\overleftrightarrow{AB}$.

on $\overline{OB}$

$y=2x$: $f(x,2x) = 2x^2 - 4x + (2x)^2 - 4(2x) + 1$

$= 6x^2 - 12x + 1$

$f'(x,2x) = 12x - 12 = 0$ giving $x = 1$, so $y = 2 \times 1 = 2$.

But $B(1,2)$ is not in the middle of $\overleftrightarrow{OB}$.

We consider $f(x,y)$ at $O(0,0)$, $A(0,2)$, and $B(1,2)$.

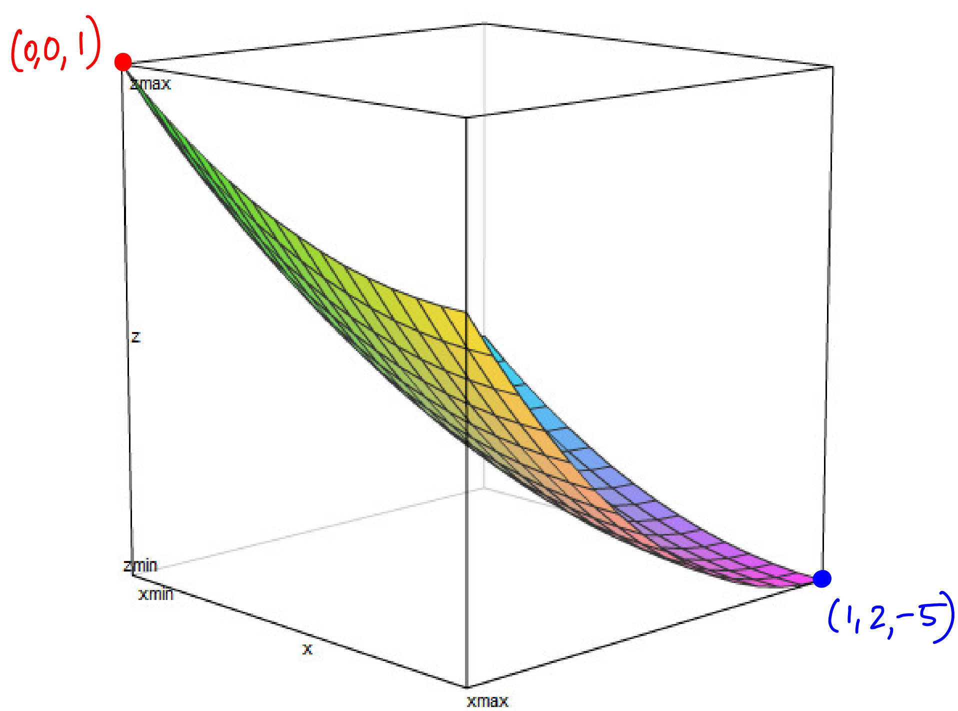
O: $f(0,0) = 1.$ ← absolute maximum in R

A: $f(0,2) = 2(0)^2 - 4(0) + (2)^2 - 4(2) + 1 = -3.$

B: $f(1,2) = 2(1)^2 - 4(1) + (2)^2 - 4(2) + 1 = -5.$ ← absolute minimum in R

The absolute maximum of f in region R occurs at $O(0,0)$, and is $f(0,0) = 1$. The absolute minimum of f in R occurs at $B(1,2)$, and is $f(1,2) = -5$.

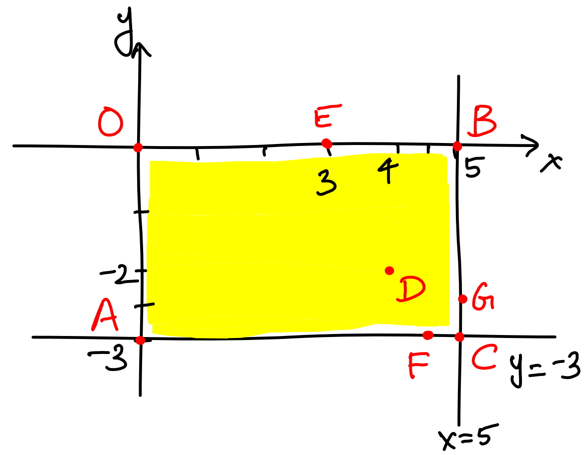
Let's visualize the function in the region R of interest. Below, we set $0 \le x \le 1$, $0 \le y \le 2$.



Notice that the absolute minimum and maximum identified are specific for the region R – they are not the global maximum and minimum over the entire domain.

35. $T(x,y) = x^2 + xy + y^2 - 6x + 2$. Find absolute maximum and absolute minimum of T on the region R that is the rectangular plate defined by $0 \le x \le 5$, $-3 \le y \le 0$.

The corner points are $O(0,0)$, $A(0,-3)$, $B(5,0)$ and $C(5,-3)$.



(i) Look for interior critical points in R.

$$T_x = 2x + y - 6 = 0 \quad \text{——— (1)}$$

$$T_y = x + 2y = 0 \quad \text{——— (2)}$$

$(2) \Rightarrow x = -2y$. So $(1) \Rightarrow$

$$-4y + y - 6 = 0 \Rightarrow -3y = 6$$

i.e., $y = -2$, So $x = -2(-2) = 4$

So $D(4,-2)$ is an interior critical point.

(ii) Examine $T(x,y)$ on boundary segments

<u>OA</u>. $x=0 \Rightarrow T(0,y) = (0)^2 + 0y + y^2 - 6(0) + 2 = y^2 + 2$

$T'(0,y) = 2y = 0$ giving $O(0,0)$, a corner point.

<u>OB</u> $y=0 \Rightarrow T(x,0) = x^2 + x(0) + (0)^2 - 6x + 2 = x^2 - 6x + 2$.

$T'(x,0) = 2x - 6 = 0 \quad x = 3$. So we add $E(3,0)$ to the list of critical points.

AC

$y=3 \Rightarrow T(x,-3) = x^2+(-3)x+(-3)^2-6x+2 = x^2-9x+11$

$T'(x,-3) = 2x-9 = 0$ giving $x=\frac{9}{2}$.

We add $F(9/2,-3)$ to the list of critical points.

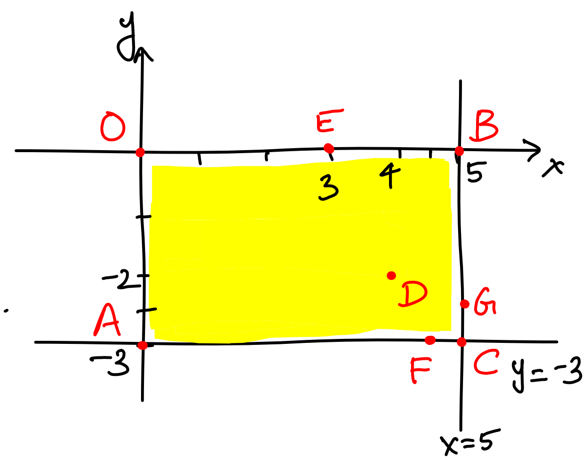
BC

$x=5 \Rightarrow T(5,y) = (5)^2+(5)y+y^2-6(5)+2 = y^2+5y-3.$

$T'(5,y) = 2y+5 = 0 \Rightarrow y = -5/2.$

We add $G(5,-5/2)$ to the list of critical points.

We compute f at the eight points (critical or corner points):
$O(0,0)$, $A(0,-3)$, $B(5,0)$, $C(5,-3)$,
$D(4,-2)$, $E(3,0)$, $F(9/2,-3)$, and $G(5,-5/2)$.



$f(x,y) = x^2+xy+y^2-6x+2$

$O: f(0,0) = 2$	$B: f(5,0) = -3$	$D(4,-2) = -10$	$F(9/2,-3) = -37/4$
$A: f(0,-3) = 11$	$C: f(5,-3) = -9$	$E(3,0) = -7$	$G(5,-5/2) = -37/4$

$A(0,-3)$ is the absolute maximum giving $f=11$, and
$D(4,-2)$ is the absolute minimum, giving $f=-10$.

in R.

Notice that we are **not** using the second derivative test to find if a critical point is a local maximum or a local minimum. Irrespective of this information, we would have to compare the value of the function at all these points. So, we do not bother to classify the critical points as local optima, saddle points, etc.

Let's visualize the function $T(x,y)$ over the region R.

$0 \le x \le 5$, $-3 \le y \le 0$ here. Here are two views.