MATH 273 – Lecture 19 (10/28/2014)

(Section 14.1)

$$\iint_R f(x,y)\,dA = \int_c^d \underbrace{\int_a^b f(x,y)\,dx}_{\text{inner}}\,dy = \int_a^b \underbrace{\int_c^d f(x,y)\,dy}_{\text{inner}}\,dx,$$

$R:\ a \le x \le b,\ c \le y \le d$

assuming $f(x,y)$ is continuous over R.

1. $$\int_1^2\int_2^4 2xy\,dy\,dx = \int_1^2 \left[x\int_2^4 2y\,dy\right]dx$$

x-limits

y-limits

book has 0 instead of 2

Recall:

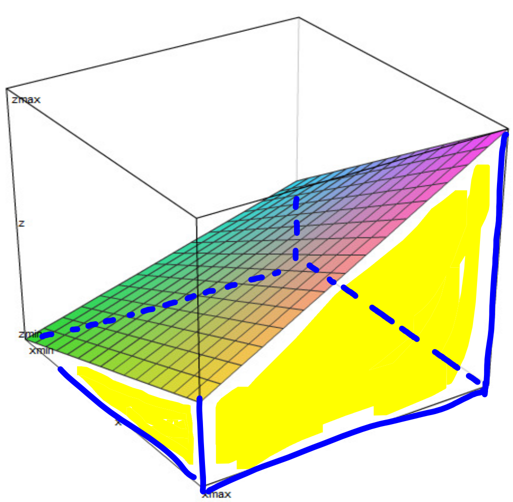
$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

outer inner

$$= \int_1^2 x\left(y^2\Big|_2^4\right)dx$$

$$= \int_1^2 x\,(\underbrace{(4)^2}_{16} - \underbrace{(2)^2}_{4})\,dx = \int_1^2 12x\,dx = 6x^2\Big|_1^2$$

$$= 6(\underbrace{(2)^2}_{4} - \underbrace{(1)^2}_{1}) = 6 \times 3 = 18.$$



The volume of the solid block under $z = 2xy$ bounded by R.

7. $$\int_0^1\int_0^1 \frac{y}{1+xy}\,dx\,dy$$

$$= \int_0^1 \left(\ln|1+xy| \Big|_0^1 \right) dy$$

$$= \int_0^1 \left(\ln|(1+y)| - \underbrace{\cancel{\ln 1}}_{=0} \right) dy = \int_0^1 \ln(|1+y|)\,dy$$

$$= (1+y)\ln|1+y| - (1+y) \Big|_0^1$$

$$= \left[(1+1)\ln(1+1) - (1+1)\right] - \left[1 \underbrace{\cancel{\ln 1}}_{=0} - 1\right]$$

$$= 2\ln 2 - 1.$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \ln x\,dx = x\ln x - x + C$$

$$\frac{d}{dx}(x\ln x - x) = \cancel{x \cdot \frac{1}{x}} + 1 \cdot \ln x - \cancel{1}$$

8. $$\int_1^4\int_0^4 \left(\frac{x}{2} + \sqrt{y}\right) dx\,dy = \int_1^4 \left(\left[\frac{1}{4}x^2 + \sqrt{y}\,x\right] \Big|_0^4 \right) dy$$

$$= \int_1^4 \left(\frac{1}{4}\left((4)^2 - (0)^2\right) + \sqrt{y}\,(4-0) \right) dy = \int_1^4 (4 + 4\sqrt{y})\,dy$$

$$= 4y + \frac{4y^{3/2}}{(3/2)} \Big|_1^4 = 4(4-1) + \frac{8}{3}\left((4)^{3/2} - (1)^{3/2}\right)$$

$$= 4 \times 3 + \frac{8}{3}(8-1) = 12 + \frac{56}{3} = \frac{92}{3}.$$

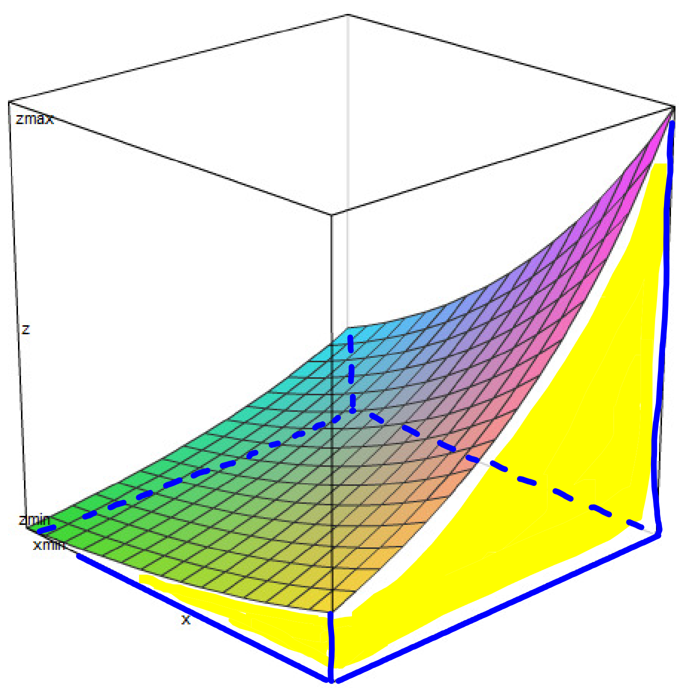
9 $$\int_0^{\ln 2}\int_1^{\ln 5} e^{2x+y}\,dy\,dx = \int_0^{\ln 2}\left(\int_1^{\ln 5}(e^{2x})e^{y}\,dy\right)dx$$

$$= \int_0^{\ln 2} e^{2x}\left(e^{y}\Big|_1^{\ln 5}\right)dx = \int_0^{\ln 2} e^{2x}\left(e^{\ln 5} - e^{1}\right)dx$$

$$= \int_0^{\ln 2} e^{2x}(5-e)\,dx = (5-e)\,\frac{1}{2}e^{2x}\Big|_0^{\ln 2}$$

$$= \left(\frac{5-e}{2}\right)\left(\underbrace{e^{2\ln 2}}_{4} - \underbrace{e^{2\times 0}}_{1}\right) = \frac{3}{2}(5-e).$$

$$\frac{d}{dx}\left(e^{2x}\right) = 2.e^{2x}$$



$0 \le x \le \ln 2$
$0 \le y \le \ln 5$ } giving
$1 \le z \le 20.$

The volume of the solid object bounded above by $z = e^{2x+y}$ and the rectangle $0 \le x \le \ln 2, 0 \le y \le \ln 5$ is $\frac{3}{2}(5-e)$.

15. Find $\iint_R xy\cos y \, dA$ for $R: -1 \le x \le 1, \ 0 \le y \le \pi$

Do integration w.r.t x first! $\longrightarrow$ The calculation simplifies this way.

$$\int_0^{\pi}\int_{-1}^{1} xy\cos y \, dx\, dy = \int_0^{\pi} y\cos y \left(\frac{1}{2}x^2 \Big|_{-1}^{1} \right) dy = \int_0^{\pi} \frac{y\cos y}{2} \left(\underbrace{\cancel{1^2 - (-1)^2}}_{1-1}^{=0} \right) dy = 0 \ !$$

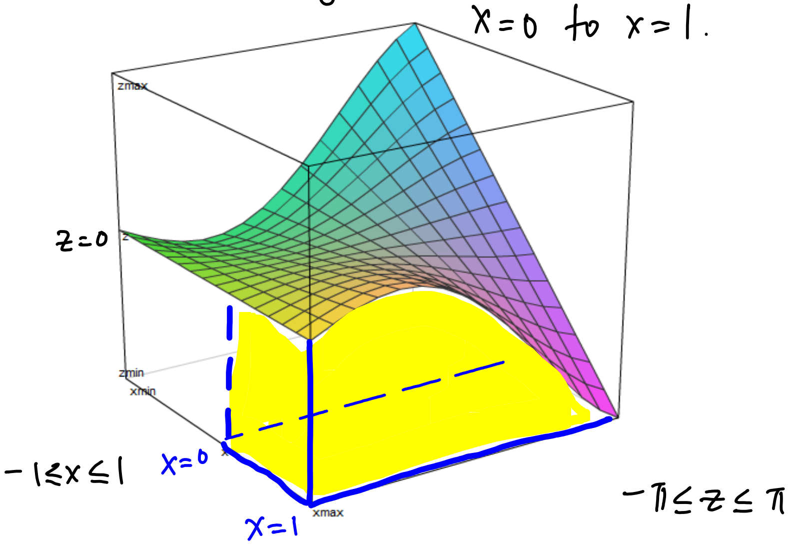
We get the same answer if we integrate w.r.t to y first.

$$\int_{-1}^{1}\int_0^{\pi} xy\cos y \, dy\, dx = \int_{-1}^{1} x\left(\int_0^{\pi} y\cos y \, dy \right) dx = \int_{-1}^{1} x \left[y\sin y + \cos y \Big|_0^{\pi} \right] dx$$

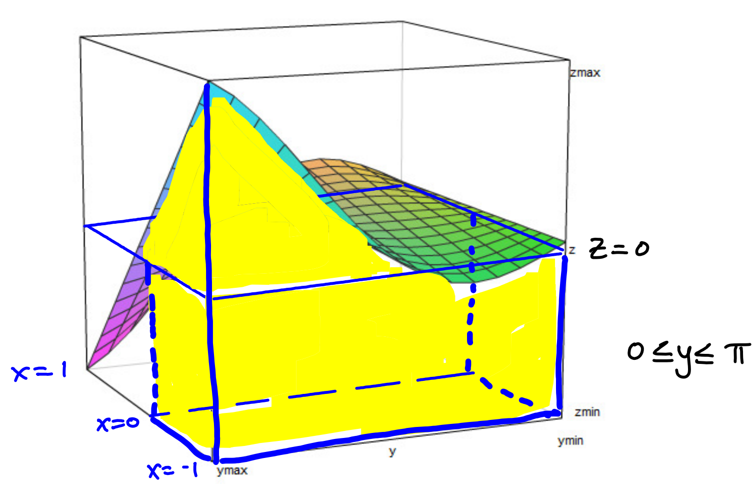
$$= \int_{-1}^{1} x \left[\underbrace{\pi \cancel{\sin}\pi}_{=0} + \underbrace{\cos\pi}_{-1} - \left(\underbrace{0\cancel{\sin} 0}_{=0} + \underbrace{\cos 0}_{1} \right) \right] dx = \int_{-1}^{1} -2x \, dx = -x^2 \Big|_{-1}^{1} = -(1^2 - (-1)^2) = 0.$$

Recall that the double integral computes the volume under the surface $z = f(x,y)$ and above the xy-plane. If the volume is **below the xy-plane**, it is **negative**. In this case, the positive and negative volumes cancel each other.

negative volume $(=-1)$ from $x=0$ to $x=1$.



positive volume $(=1)$ from $x=-1$ to $x=0$.



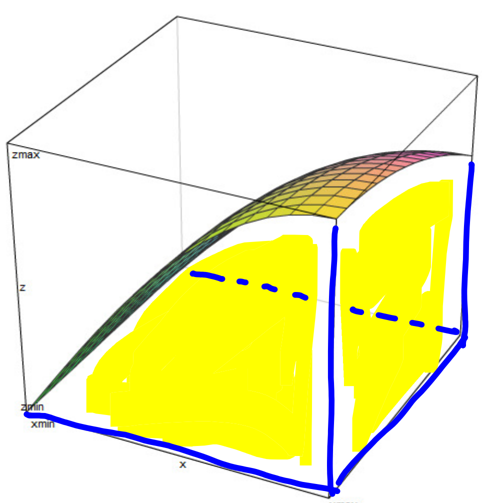
27. Find volume of region bounded above by the surface $z = 2\sin x \cos y$ and below by rectangle R $0 \le x \le \pi/2$, $0 \le y \le \pi/4$.

$$\text{Volume} = \iint_R 2\sin x \cos y \, dA$$

$$= \int_0^{\pi/4}\int_0^{\pi/2} 2\sin x \cos y \, dx\, dy = \int_0^{\pi/4}\left[2\cos y\,(-\cos x)\Big|_0^{\pi/2}\right] dy$$

$$= \int_0^{\pi/4} 2\cos y\Big(\underbrace{-\cos \pi/2}_{=0} - (\underbrace{-\cos 0}_{1})\Big)\, dy = \int_0^{\pi/4} 2\cos y \, dy$$

$$= 2\sin y\Big|_0^{\pi/4} = 2\Big(\underbrace{\sin \pi/4}_{=\frac{1}{\sqrt{2}}} - \underbrace{\sin 0}_{=0}\Big) = \frac{2}{\sqrt{2}} = \sqrt{2}.$$



$0 \le x \le \pi/2$, $0 \le y \le \frac{\pi}{4}$ here.