MATH 273 – Lecture 21 (11/04/2014)

Exam 2: Next Thursday (Nov 13)
- Practice Exam 2 will be posted.

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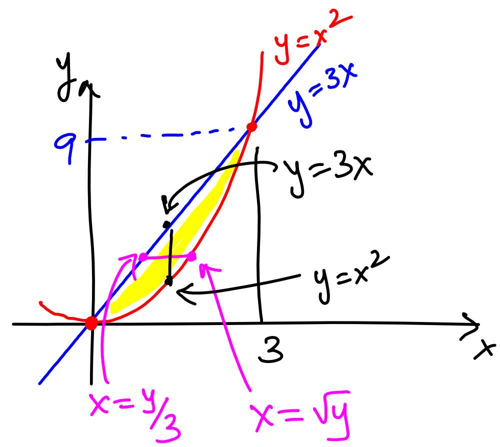
11. Write an integral for $\iint_R dA$ over region R using (a) vertical cross sections, and (b) horizontal cross section.

$y=3x$, $y=x^2$ $x^2 = 3x$ $x^2-3x=0 \Rightarrow x(x-3)=0$

points of intersection of these two curves

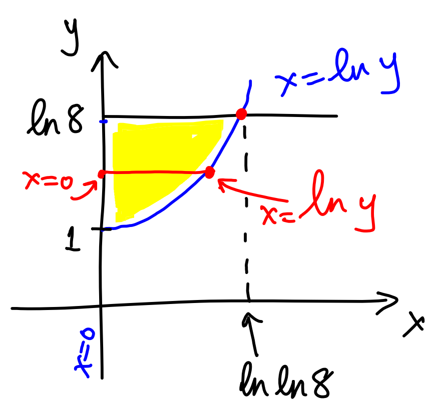
gives $x=0$, $x=3$, for which $y=0$, $y=9$, i.e., the points are $(0,0)$ and $(3,9)$.



(a) $\displaystyle\int_0^3\int_{x^2}^{3x} dy\,dx$ (b) $\displaystyle\int_0^9\int_{y/3}^{\sqrt{y}} dx\,dy$

21. Sketch the region of integration, and evaluate the integral.

$$\int_1^{\ln 8}\int_0^{\ln y} e^{x+y}\,dx\,dy$$



plot $x=\ln y$, i.e., $y=e^x$

The right point of intersection has $x=\ln(\ln 8) = \ln\ln 8$.

$$\int_1^{\ln 8}\int_0^{\ln y} e^{x+y}\,dx\,dy = \int_1^{\ln 8} e^y\left(\int_0^{\ln y} e^x\,dx\right)dy = \int_1^{\ln 8}\left(e^{y+x}\Big|_0^{\ln y}\right)dy$$

$$= \int_1^{\ln 8}\left(e^{y+\ln y} - e^{y+0}\right)dy = \int_1^{\ln 8}\left(e^y\cdot \underbrace{e^{\ln y}}_{y} - e^y\right)dy$$

$$= \int_1^{\ln 8}\left(ye^y - e^y\right)dy = \left[\underbrace{ye^y - e^y}_{\frac{d}{dy}(ye^y - e^y) = ye^y} - e^y\right]\Bigg|_1^{\ln 8}$$

$$= \left(\ln 8\, e^{\ln 8} - 2e^{\ln 8}\right) - \left(e^1 - 2e^1\right)$$

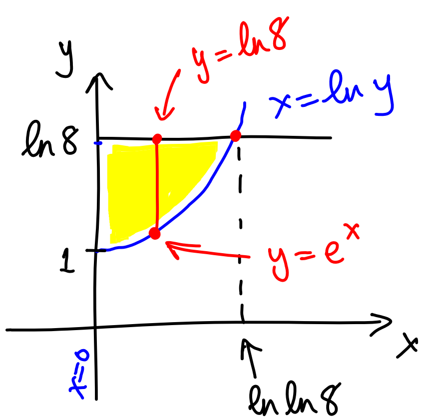
$$= 8\ln 8 - 16 + e = e + 8(\ln 8 - 2).$$

$$\frac{d(uv)}{dy} = u\frac{dv}{dy} + v\frac{du}{dy}$$

or

$$d(uv) = u\,dv + v\,du$$

Let's evaluate the integral now by reversing the order of integration.



$$\int_0^{\ln\ln 8}\int_{e^x}^{\ln 8} e^{x+y}\,dy\,dx = \int_0^{\ln\ln 8}\left(e^{x+y}\Big|_{e^x}^{\ln 8}\right)dx$$

$$= \int_0^{\ln\ln 8}\left(e^{x+\ln 8} - e^{x+e^x}\right)dx$$

$$= \int_0^{\ln\ln 8}\left(8e^x - e^x\cdot e^{e^x}\right)dx$$

$$= \int_0^{\ln\ln 8} \left(8e^x - e^x \cdot e^{e^x}\right) dx = 8e^x - e^{e^x} \Big|_0^{\ln\ln 8}$$

recall: $\frac{d}{dx}\left(e^{f(x)}\right) = e^{f(x)} \cdot f'(x)$

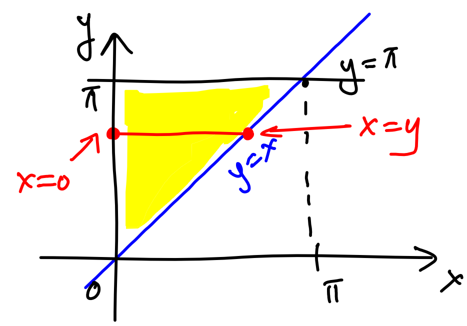
$$= \left(8e^{\ln\ln 8} - e^{e^{\ln\ln 8}}\right) - \left(8e^0 - e^{e^0}\right)$$

$e^{\ln 8} = 8$

$$= 8\ln 8 - 8 - 8 + e = e + 8(\ln 8 - 2).$$

47. Sketch region of integration, reverse the order of integration, and evaluate the integral.

$$I = \int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y}\, dy\, dx$$



Originally, vertical cross sections are used. We reverse to use horizontal cross sections.

$$I = \int_0^{\pi}\int_0^{y} \frac{\sin y}{y}\, dx\, dy$$

$$= \int_0^{\pi}\left(\frac{\sin y}{y}\, x\Big|_0^y\right) dy = \int_0^{\pi}\left(\frac{\sin y}{y}(y-0)\right) dy = \int_0^{\pi} \sin y\, dy$$

$$= -\cos y\Big|_0^{\pi} = 1 - {}^-1 = 2.$$

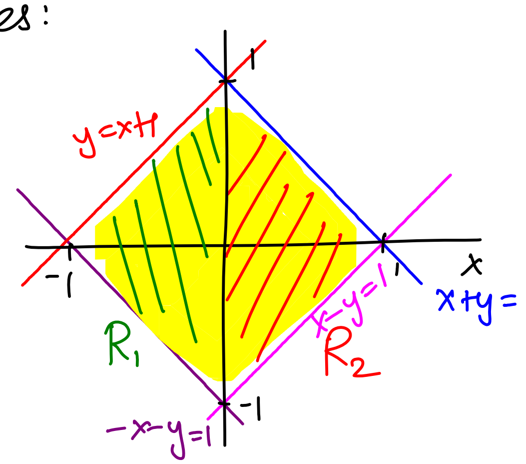
In all the integrals we have seen so far, the region of integration R is bounded essentially by two curves. Notice that even in the case where R is a triangle, as seen in the example above, $y=x$ and $y=\pi$ were sufficient to describe it, along with $x=0$. Now we consider more general regions R, which we split into component regions R_1, R_2, R_3, etc., where each component region is simpler, just as we have seen so far.

55. Find $I = \iint\limits_R (y-2x^2)\,dA$ where R is the region bounded by the square $|x|+|y|=1$.

$|x|+|y|=1$ splits into four lines:

$\pm x \pm y = 1$, i.e.,

$$x+y=1$$
$$x-y=1$$
$$-x+y=1$$
$$-x-y=1$$



The region is bounded by 4 curves, instead of 2. So split into two regions bounded by two curves each.

... we'll finish this problem in the next lecture ...