Principles of Optimization (Fall 2024)
Final Examination

- There are **six** problems in this exam, presented in pages 2–3.

- The total points (given in parentheses) add to 100.

- This is a **LIMITED open resources** exam. You **are allowed** to use **class resources** such as lecture notes, homework and exam solutions, handouts, and AMPL files—anything that is **available on the course web page**. You can also **use AMPL**, if needed.

- You are **NOT PERMITTED** to

    - use AI engines such as ChatGPT, CoPilot, etc.,
    - search the internet for the exam (except the course web page),
    - communicate with anyone else on the exam,
    - post on online forums about the exam, or
    - use any *other* textbook, handouts, or online resources.

- You **must start your exam** by writing down the following statement word-by-word, and signing under the same.

    > I promise that I will not seek external help from any person, AI, or the internet while working on this exam. I will use only the resources posted on the class web page while working on the same.
    > —*Signature*

- You **must end your exam** by writing down the following **second** statement word-by-word, and again signing under the same.

    > As promised, I did not use any help from another person or through online or email communications or from AI engines while working on this exam.
    > —*Signature*

- You **must email your submission as a SINGLE PDF file** to kbala@wsu.edu. You are welcome to write answers by hand, and scan the writings.

- Your **file name should identify you** in the usual manner. If you are Scott Tenorman, you should name your submission ScottTenorman_Final.pdf (and **NOT** Scott_Tenorman*.pdf or ScotTenorman*.pdf or ...). You could add anything more to your filename *after* these terms, e.g., ScottTenorman_Final_Math364.pdf. Please avoid white spaces in the file name :-).

- **Begin the SUBJECT of your email submission with the same `FirstnameLastname`, e.g., "ScottTenorman Final Exam submission".**

- This exam must be emailed to me **by 10:00 PM on Thursday, December 12, 2024**.

1. (13) The optimal tableau of an LP with two "$\leq$" constraints is given below (s_1, s_2 are the slack variables for these two constraints).

z	x_1	x_2	x_3	s_1	s_2	rhs
1	0	-2	0	-3	-2	-35
0	0	$1/4$	1	$1/2$	0	$5/2$
0	1	$-1/2$	0	$-1/6$	$1/3$	$5/2$

Give the initial (starting) tableau.

2. An LP and its optimal tableau are given below.

$$\begin{aligned}
\min \quad z &= 4x_1 + x_2 \\
\text{s.t.} \quad 4x_1 + 3x_2 &\geq 6 \\
x_1 + 2x_2 &\leq 3 \\
3x_1 + x_2 &= 3 \\
x_1, \ x_2 &\geq 0
\end{aligned}$$

z	x_1	x_2	e_1	s_2	a_1	a_3	rhs
1	a	e	j	$1/5$	M	$M - (7/5)$	n
0	b	f	k	$3/5$	0	$-1/5$	p
0	c	g	ℓ	$-1/5$	0	$2/5$	q
0	d	h	m	1	-1	1	r

(a) (9) Find the values of the 16 unknowns $a, b, c, d, e, f, g, h, j, k, \ell, m, n, p, q$, and r. Explain your method. You must **not** perform all the steps of the big-M simplex method to find these values.

(b) (6) Find the dual of the given LP and its optimal solution.

(c) (6) Find the range of values of the right-hand side of the *third* constraint for which the current basis remains optimal.

(d) (4) Find the new optimal solution and new optimal objective function value if the right-hand side of the *second* constraint is changed to 5.

3. (17) Columbia Records has contracted with Taylor Swift to record an album with ten songs. The duration of the songs are $8, 5, 4, 5, 7, 10, 7, 9, 12$, and 8 minutes, respectively. Columbia wants to release the album in two web releases, with some songs as well as extra bonus material included in both releases. Each release has a limit of 40 minutes for the songs. Columbia wants to distribute the ten songs among the two releases such that the difference in total duration of the songs on each release is minimized. In addition, songs 3 and 4 cannot be included on the same release. Because of their long durations, if songs 6 and 9 are on the same release, then song 8 cannot be on that release. Formulate an integer program to distribute the songs to the two releases of the album.

4. (16) Consider the following two LPs.

$$
\begin{array}{rlrl}
\max \ z = & c_1 x_1 + c_2 x_2 & & \\
\text{s.t.} & a_{11} x_1 + a_{12} x_2 & \leq & b_1 \\
& a_{21} x_1 + a_{22} x_2 & \leq & b_2 \\
& x_1, \ x_2 & \geq & 0
\end{array}
\tag{LP 1}
$$

$$
\begin{array}{rlrl}
\max \ z = & (1/25)\, c_1 x_1 + c_2 x_2 & & \\
\text{s.t.} & a_{11} x_1 + 25 a_{12} x_2 & \leq & 10 b_1 \\
& a_{21} x_1 + 25 a_{22} x_2 & \leq & 10 b_2 \\
& x_1, \ x_2 & \geq & 0
\end{array}
\tag{LP 2}
$$

Suppose that $\{x_1, x_2\}$ is the optimal basis for both the LPs. Let the optimal solution for LP 1 be $x_1 = 100$, $x_2 = 250$ with $z^* = 1050$. Also, suppose that for LP 1, the shadow price of constraint 1 is 200 and that of constraint 2 is 100. Find the optimal solution to LP 2, its optimal objective function value, and the optimal solution to its dual.

5. (17) Model the statement using extra binary variables and big-M, when x and y are integers:

$$\textbf{either} \ \ x + 2y \neq 2 \ \ \textbf{or} \ \ 2x + 3y \geq 5.$$

6. (12) Use complementary slackness conditions and the dual theorem to find the optimal solution to the following LP:

$$
\begin{array}{rlrl}
\min \ z = & 6x_1 + 14x_2 + 8x_3 & & \\
\text{s.t.} & 3x_1 + 2x_2 + 4x_3 & \geq & 8 \\
& 2x_1 + 7x_2 + 2x_3 & \geq & 10 \\
& x_1, \ x_2, \ x_3 & \geq & 0
\end{array}
$$