Principles of Optimization (Fall 2024)
Midterm Examination

Name: _____ WSU ID: _____

- There are **six** problems in this exam.
- The total points (given in parentheses) add to 105. You will be graded for 100.
- Space given here should be sufficient to do all necessary work, but in case you use any extra sheets of paper for rough work, write your name on each such sheet, and attach *all* such sheets to this exam before submitting.
- *Time is precious*—try to first finish problems that you are sure of. Good luck!

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1. (18) Suppose we have the following tableau in canonical form for a maximization LP.

z	x_1	x_2	x_3	x_4	x_5	x_6	rhs
1	0	0	0	c_1	c_2	c_3	z^*
0	0	-2	a_3	a_5	-1	0	0
0	1	a_1	0	-3	0	a_7	2
0	0	a_2	a_4	-4	a_6	a_8	b

   State conditions on $a_1, \ldots, a_8,\ b,\ c_1, c_2, c_3$ and z^* that make the following statements true:

   (a) The current bfs is optimal, and there are alternative optimal solutions, but *no* alternative optimal bfs's.

   (b) The current tableau is feasible, but not optimal, and x_4 can enter the basis. In this case, what will be the value of z^* after pivoting x_4 into the basis?

2. (17) Solve the following LP graphically.

$$\begin{array}{lrcl} \min\ z = & 2x_1 + x_2 & & \\ \text{s.t.} & x_1 + x_2 & \geq & 2 \\ & 3x_1 + x_2 & \geq & 4 \\ & 3x_1 + 2x_2 & \leq & 10 \\ & x_1 \text{ urs}, \quad x_2 & \geq & 0 \end{array}$$

3. (17) Use the tableau simplex method to solve the following LP:

$$\begin{array}{lrcl} \max\ z = & 3x_1 - 2x_2 + x_3 & & \\ \text{s.t.} & x_1 + 5x_2 - x_3 & \leq & 4 \\ & 2x_1 - 2x_2 + 4x_3 & \leq & 6 \\ & x_1,\ x_2,\ x_3 & \geq & 0 \end{array}$$

4. (17) Kyle gave Cartman the tableau of a max-LP to solve using the simplex method. The LP had two variables (x_1, x_2) and two $\leq$ constraints. Cartman decided to play a joke on Kyle, so he

   (a) changed the coefficient of x_1 in the first constraint from 6 to 9, and

   (b) solved the LP as a *min*-LP.

   With these modifications, Cartman got the following "optimal" tableau after performing a single pivot. Find the correct optimal tableau that Kyle should have obtained.

z	x_1	x_2	s_1	s_2	rhs
1	0	-2	$-1/3$	0	2
0	1	0	$1/9$	0	$2/3$
0	0	2	$-5/9$	1	$2/3$

5. (18) State whether each of the following statements is *True* or *False*. Justify your answer.

   (a) The feasible region of an LP is bounded only if all the variables are nonnegative.

   (b) The value of a basic variable in an optimal simplex tableau is always nonzero.

   (c) Let there be two non-basic variables that could both enter the basis, and both improve the objective function at the same rate. Then the LP has alternative optimal bfs's.

6. (18) The Child's Play Manufacturing Company can produce its marvelous toy, Ugly Chucky, on one of three shifts: regular shift, overtime shift, and a burn-midnight-oil shift. For the next three production periods, it has collected the following data:

period	regular time capacity	overtime capacity	demand
1	5000	1000	5400
2	5500	1100	6100
3	3200	950	6000

   The burn-midnight-oil capacity is 500 units in each of the three production periods. The initial inventory at the beginning of period 1 is 225 units. The inventory at the end of each period must be between 150 and 250 units. The unit production cost is 10 cents on regular time, 14 cents on overtime, and 25 cents on burn-midnight-oil time. The cost of storing a unit from one period to the next is 4 cents. Formulate an LP that minimizes the production and inventory costs of Child's Play.