Principles of Optimization (Fall 2024)
Practice Midterm Examination

Name: _____   WSU ID: _____

- There are **six** problems in this exam.

- The total points (given in parentheses) add to 105. You will be graded for 100.

- Space given here should be sufficient to do all necessary work, but in case you use any extra sheets of paper for rough work, write your name on each such sheet, and attach *all* such sheets to this exam before submitting.

- *Time is precious*—try to first finish problems that you are sure of. Good luck!

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1. (18)   Suppose we have obtained the following tableau for a minimization LP.

z	x_1	x_2	x_3	x_4	x_5	x_6	rhs
1	c_1	c_2	0	c_3	c_4	c_5	z^*
0	3	a_1	1	0	a_2	a_3	1
0	-1	-2	0	a_4	-1	a_5	b
0	a_6	-4	0	0	-3	a_7	3

State conditions on $a_1, \ldots, a_7$, $c_1, \ldots, c_5$, b, and z^* required to make the following statements true.

   (a) The current solution is optimal, and there are alternative optimal basic feasible solutions.

   (b) The LP is unbounded.

2. (18)   Solve the following LP using tableau simplex method.

$$\begin{array}{rrcl} \max \ z = & 2x_1 - \ x_2 + 2x_3 & & \\ \text{s.t.} & 3x_1 + 2x_2 + \ x_3 & \leq & 6 \\ & -4x_2 + 2x_3 & \leq & 2 \\ & x_1, \quad x_2, \quad x_3 & \geq & 0 \end{array}$$

3. (18)   Solve the following LP graphically.

$$\begin{array}{rrcl} \max \ z = & 3x_1 - 2x_2 & & \\ \text{s.t.} & 2x_1 + \ x_2 & \leq & 6 \\ & 2x_1 + 5x_2 & \geq & 6 \\ & -2x_1 + 3x_2 & \leq & 10 \end{array}$$

4. (18)   Kyle just solved an LP with two constraints. Cartman wants to solve the same LP, but Kyle would not give it to him. After some pleading by Cartman, Kyle agrees to share the optimal tableau with him, which is given below. He further confirms that the variables s_1, s_2 are the slack variables for the two constraints in the LP. Find the original LP for Cartman.

z	x_1	x_2	x_3	s_1	s_2	rhs
1	0	-3	0	-3	$-1/2$	-20
0	1	-1	0	1	-1	2
0	0	2	1	0	$1/2$	2

5. (15)   State whether each of the following statements is *True* or *False*. Justify your answer.

   (a) If the feasible region of an LP has no corner points, then the LP must be unbounded.

   (b) If the coefficient of a nonbasic variable in Row-0 of a max-LP is strictly less than zero, pivoting the variable in to the basis will strictly increase the objective function value.

   (c) An equality constraint could be nonbinding at a feasible solution that is not a bfs.

6. (18)   Wendy Testaburger is the CEO of South Park Loading Company, which needs to lease warehouse space over the next 3 months. The space requirement in thousands of square feet are 15, 13, and 17, for Months 1,2, and 3, respectively. The monthly leasing costs (in dollars per thousand square feet per month) for various lengths of the lease are 280, 245, and 220, for 1-, 2-, and 3-month leases, respectively. Wendy can lease any amount of space for any length she wants, as long as the whole lease falls within the three-month period (so, she cannot take a 2-month lease at the start of the third month, for instance). She could start two or more leases of different lengths at the same time. Formulate an LP to figure out the leasing schedule which minimizes the total costs Wendy will incur.