Principles of Optimization (Fall 2024): Homework 10

- There are **four problems**, and the total points (given in parentheses) add up to 125. You will be graded for 120 points (with the possibility of getting up to 5 points as extra credit).
- **You must submit your homework by email as follows:**
  - **You must email your submission as a PDF file to kbala@wsu.edu**. You are welcome to write answers by hand, and scan the writings (or take pictures of your writings) into a **PDF file**.
  - **Your file name should identify you in this manner: If you are Officer Barbrady, say, you should name your submission OfficerBarbrady_Math364_Hw10.pdf. Please avoid white spaces in the file name (use "_" or "-" instead).**
  - **Begin the SUBJECT of your email submission with the same `FirstnameLastname` expression, e.g., "OfficerBarbrady Math364 Hw10 submission".**
- **This homework is due by 5:00 PM on Thursday, November 14.**

1. (25) Find the dual of the following LP.

$$\begin{array}{lllll} \max\ w = & 3.4y_1 + 2y_3 - 1.5s_4 & & \\ \text{s.t.} & 2x_2 + 3y_3 - 7s_4 & \geq & 4.6 \\ & -y_1 + 3x_2 - 0.5s_4 & = & 1 \\ & 5y_1 + 2x_2 + 5y_3 + s_4 & \leq & 7 \\ & -2y_1 + x_2 - 4y_3 + 3s_4 & \leq & 3 \\ & y_1 \leq 0,\ x_2, y_3 \geq 0,\ s_4 \text{ urs} & & \end{array}$$

2. (30) This problem asks you to verify on an example that the dual of a general (i.e., not normal) LP written down directly (e.g., using the table of primal-dual relationships) follows the same rules (in an equivalent fashion) as followed by the dual of a normal LP. Hence the two dual LPs are equivalent.

   (a) Write the dual (D) of the following primal LP denoted (P).

$$\begin{array}{lllll} \max\ z = & 2x_1 + 3x_2 & & \\ \text{s.t.} & x_1 + 4x_2 & = & 2 \\ & 3x_1 - x_2 & \geq & 1 \\ & x_1 \geq 0, x_2 \leq 0 & & \end{array} \qquad \text{(P)}$$

   (b) Notice (P) is not a normal max-LP. Convert (P) to the the equivalent normal max-LP (P'). Write the dual (D') of (P'). Thus, (D') will be a normal-min LP. *Hint: An equation can be replaced by the two corresponding inequalities with the senses going both ways, e.g., $x = 4$ is equivalent to $x \leq 4$ and $x \geq 4$.*

   (c) Show that the dual LP (D') is equivalent to the original dual (D), after appropriate modifications of the variable definitions in (D').

3. (30) Write the dual of the Chukees Toys LP (Problem 1 in Homework 2). Then write an economic interpretation of the dual LP. You should give an interpretation for each variable, the objective function, and each constraint in the dual LP.

4. (40) Write the dual of the NYPD Cops staffing problem (Problem 1 in Homework 3). Then write an economic interpretation of the dual LP. You should write an interpretation for the objective function, *at least one* variable in the dual (corresponding to a (shift i demand) constraint), and for *at least one* constraint in the dual LP corresponding to a consecutive shifts variable in the primal (of the form $x_{i,i+1}$) as well as *at least* one constraint corresponding to a non-consecutive shifts variable in the primal LP (of the form x_{ij} for $j > i + 1$).