Principles of Optimization (Fall 2024): Homework 3

- There are five problems, and the total points (given in parentheses) add up to 160. You will be graded for 150 points (with the possibility of getting up to 10 points as extra credit).
- When writing LP formulations, make sure you
  - clearly define the decision variables;
  - write the objective function and constraints as linear functions or (in)equalities;
  - add appropriate sign restrictions for the variables (e.g., non-negativity); and
  - add brief interpretations (in braces) for each constraint as well as the objective function.
- **You must submit your homework by email as follows:**
  - **You must email your submission as a PDF file to kbala@wsu.edu**. You are welcome to write answers by hand, and scan the writings (or take pictures of your writings) into a **PDF file**.
  - **Your file name should identify you in this manner: If you are Bebe Stevens, say, you should name your submission BebeStevens_Math364_Hw3.pdf. Please avoid white spaces in the file name (use "_" or "-" instead).**
  - **Begin the SUBJECT of your email submission with the same `FirstnameLastname`, expression, e.g., "BebeStevens Math364 Hw3 submission".**
  - **This homework is due by 5:00 PM on Thursday, September 12.**

1. (30) Cops at the Nice York Police Department (NYPD) work two 4-hour shifts each day from the six shifts: 12–4 AM, 4–8 AM, 8 AM–12 PM, 12–4 PM, 4–8 PM, and 8 PM–12 AM. The numbers of cops needed for each of the six shifts in NYPD are 8, 7, 12, 9, 6, and 10 (in the order of the shifts specified above). If a cop works two consecutive shifts, their pay is \$15 per hour; if the two shifts are not consecutive, their pay is \$18 per hour. Formulate an LP to minimize the total cost of meeting the daily demands for cops in NYPD. Assume the cops do *not* split their two shifts across two days, i.e., plan for just a single day.

2. (30) Killer Drugs makes the drug Doozey by mixing four chemicals. There are three active ingredients A, B, and C in Doozey. The cost per lb as well as the amounts (in lbs) of the ingredients A, B, and C in one lb of each chemical is given below. One lb of Doozey must contain at least 0.08 lbs of A, 0.10 lbs of B, and 0.07 lbs of C. Killer must produce 900 lbs of Doozey in one batch, and are required to use at least 200 lbs of chemical 3, while at most 500 lbs of chemical 4 can be used. Formulate an LP to minimize the cost to Killer Drugs for producing one batch of Doozey.

Chemical	Cost	A	B	C
1	9	0.04	0.08	0.03
2	12	0.07	0.11	0.09
3	17	0.14	0.09	0.12
4	8	0.04	0.05	0.11

3. (30) The firm Cascade produces three products A, B, and C, which could be sold at \$15, \$60, and \$105 per unit, respectively. Producing one unit of A requires 2 hours of labor; one unit of B requires 3 hours of labor and 1 unit of A; and one unit of C requires 1 hour of labor and 2 units of B. Units of A used for making B, and those of B used for making C cannot be sold separately. A total of 50 hours of labor are available. Formulate an LP to maximize Cascade's revenue.

4. (35) Spoiler Inc. must meet the following demands for the product RotFast in the four quarters in 2024: 30, 28, 40, and 35. In each quarter, up to 25 units of RotFast could be made using regular time labor at the cost of \$50 per unit, and as many units using overtime labor at the cost of \$70 per unit. 15% of all units produced in a quarter spoil, and hence cannot be used to meet the demand. Also, at the end of each quarter, 8% of the units remaining rot, and cannot be used to meet future demands. The storage cost is \$12 per unit (assessed on only the unspoiled units remaining at the end of a quarter). Spoiler has 12 units available at the start of the first quarter, and would like to have no units left unused at the end of the fourth quarter. Formulate an LP to minimize Spoiler's total costs for meeting their demands for 2024.

5. (35) Corny Rice buys and sells rice. At the start of 2024, she has 70 tons of rice and \$1,500. At the start of each of four quarters in 2024, Corny can buy rice at the rates per ton of \$350, \$320, \$400, and \$450. At the end of each of the four quarters, Corny can sell the rice at the following prices per ton: \$300, \$400, \$380, and \$525. Corny could store at most 150 tons of rice at any time. Formulate an LP to maximize the total cash Corny will have at the end of 2024 (i.e., at the end of the fourth quarter).