Principles of Optimization (Fall 2024): Homework 6

- There are six problems, and the total points (given in parentheses) add up to 145. You will be graded for 135 points (with the possibility of getting up to 10 points as extra credit).
- **You must submit your homework by email as follows:**
  - **You must email your submission as a PDF file to kbala@wsu.edu**. You are welcome to write answers by hand, and scan the writings (or take pictures of your writings) into a **PDF file**.
  - **Your file name should identify you in this manner: If you are Uncle Jimbo, say, you should name your submission UncleJimbo_Math364_Hw6.pdf. Please avoid white spaces in the file name (use "_" or "-" instead).**
  - **Begin the SUBJECT of your email submission with the same `FirstnameLastname`, expression, e.g., "UncleJimbo Math364 Hw6 submission".**
  - **This homework is due by 5:00 PM on Thursday, October 3.**

1. (20) Solve the following LP using the simplex method.

$$\begin{array}{llcl} \min\ z = & -5x_1 - 3x_2 & & \\ \text{s.t.} & 4x_1 + 2x_2 & \leq & 12 \\ & -2x_1 - 5x_2 & \leq & 6 \\ & x_1,\ x_2 & \geq & 0 \end{array}$$

2. (25) How many optimal basic feasible solutions does the following LP have? And how many optimal solutions does it have?

$$\begin{array}{llcl} \max\ z = & 3x_1 + 3x_2 & & \\ \text{s.t.} & x_1 + \ x_2 & \leq & 6 \\ & 2x_1 + \ x_2 & \leq & 13 \\ & x_1,\ x_2 & \geq & 0 \end{array}$$

3. (30) Consider the following optimal tableau. Does the LP have more than one optimal bfs? How many optimal solutions does it have?

z	x_1	x_2	x_3	x_4	rhs
1	0	0	0	4	5
0	−2	0	1	2	3
0	−3	1	0	1	1

4. (20) Consider the following tableau for a max LP. Argue why the LP is unbounded, even though x_1 could enter the basis. You should **not** pivot x_1 into the basis to arrive at this conclusion.

z	x_1	x_2	x_3	x_4	rhs
1	-5	0	0	-1	5
0	2	0	1	0	4
0	3	1	0	-2	6

5. (30) This problem illustrates that one could get more than *two* basic feasible solutions (*bfs*'s) that are optimal. Use simplex method to describe *all* the optimal solutions of the following LP.

$$\begin{array}{lrcl} \max\ z = & 2x_1 + 3x_2 + 5x_3 + 4x_4 & & \\ \text{s.t.} & x_1 + 2x_2 + 3x_3 + x_4 & \leq & 5 \\ & x_1 + x_2 + 2x_3 + 3x_4 & \leq & 3 \\ & x_1,\ x_2,\ x_3,\ x_4 & \geq & 0 \end{array}$$

6. (20) We are given the starting tableau (Table 1) and the current tableau (Table 2) of an LP here. Find the values of all the unknowns $a, b, c, d, e, f, g, h, i, j, k$, and l.

Table 1: Starting tableau

z	x_1	x_2	x_3	x_4	x_5	rhs
1	a	1	-3	0	0	0
0	b	c	d	1	0	6
0	-1	2	e	0	1	1

Table 2: Current tableau

z	x_1	x_2	x_3	x_4	x_5	rhs
1	0	$-\frac{1}{3}$	j	k	l	-4
0	g	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	0	f
0	h	i	$-\frac{1}{3}$	$\frac{1}{3}$	1	3