Principles of Optimization (Fall 2024): Homework 8

- There are four problems, and the total points (given in parentheses) add up to 140. You will be graded for 135 points (with the possibility of getting up to 5 points as extra credit).
- **You must submit your homework by email as follows:**
  - **You must email your submission as a PDF file to kbala@wsu.edu**. You are welcome to write answers by hand, and scan the writings (or take pictures of your writings) into a **PDF file**.
  - **Your file name should identify you in this manner: If you are Bradley Biggle, say, you should name your submission BradleyBiggle_Math364_Hw8.pdf. Please avoid white spaces in the file name (use "_" or "-" instead).**
  - **Begin the SUBJECT of your email submission with the same `FirstnameLastname` expression, e.g., "BradleyBiggle Math364 Hw8 submission".**
- **This homework is due by 5:00 PM on Thursday, October 31.**

1. (25) How would you use linear programming to solve the following optimization problem? Notice that $|x|$ denotes the absolute value of x.

$$\begin{array}{lrcl} \max\ z = & |3x_2 - 4x_1| & & \\ \text{s.t.} & 6x_1 + 2x_2 & \leq & 7 \\ & 3x_1 + 4x_2 & \leq & 4 \\ & x_1,\ x_2 & \geq & 0 \end{array}$$

2. (30) Recall how we model an urs (unrestricted in sign) variable x_i by replacing each occurrence of x_i with $x_i^+ - x_i^-$, where $x_i^+,\ x_i^- \geq 0$. Show that the columns of x_i^+ and x_i^- in the simplex tableau are always negatives of each other, after any number of pivot operations. *Hint: Consider the effect of valid EROs on the columns of x_i^+ and x_i^-, and show that this property is maintained by each such ERO.*

3. (25) We are solving a max-LP using tableau simplex method. Suppose the variable x_ℓ is about to leave the basis.
   (a) Show that the coefficient of x_ℓ is Row-0 cannot be less than zero after the pivot is performed.
   (b) Explain why the variable x_ℓ, which has just left the basis in a pivot, cannot reenter the basis on the next pivot.

4. (60) Consider the Gaseous Chemicals LP given as Problem 3 in Homework 2. Recall the optimal solution was to run Processes 1 and 2 for 3 and 2 hours, respectively (see Homework 2 Solutions.
   (a) Find the range of values of the cost (per hour) of running Process 1 for which the current solution remains optimal. Repeat the same analysis for the cost of running Process 2.
   (b) Find the range of values of the demand for the number of units of chemical A (originally set at 10) for which the current basis remains optimal. Repeat the same analysis for the demands of chemicals B and C.
   (c) Find the shadow price of each demand constraint (i.e., for A, B, and C).
   (d) Find the new optimal solution and the new minimum cost when the demand for Chemical C is 4. You should **not** re-solve the LP from scratch here.