# MATH 364: Lecture 16 (10/10/2024)

Today: * more AMPL
* sensitivity analysis

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Offer on midterm:

* If you get $\geq 92\%$ in final, final score will replace mid-term score.
* If you get $\geq 85\%$ ($< 92\%$), the weights for final will be 30%, and mid-term = 10%.

Updates: * Hw 7 will be due Tuesday, Oct 22.
* Final exam will be take-home open book (but NO AI tools allowed).

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## More AMPL

Chukee problem: We could consider a generalization where there are multiple types of toys and multiple types of operations to make the toys.

| Toy ↓ | Assembly | Paint | .... |
|---|---|---|---|
| Dirty | 1500 | 800 | |
| Ugly | 1200 | 700 | |
| ⋮ | . | | |

See the course web page for details.

# Sensitivity Analysis

How do changes in parameters affect the optimal solution?

After solving the LP, say, you realize one of the objective function coefficients is changed by a little bit, but the rest of the problem remains the same. Could we find the new optimal solution quickly from the previous optimal solution, without re-solving the changed LP from scratch?

More generally, we want to study how sensitive the optimal solution and the optimal basis are to changes in the data of the problem. Just as we did when developing the simplex method to solve LPs, we will first study sensitivity analysis in 2D using the graphical method.

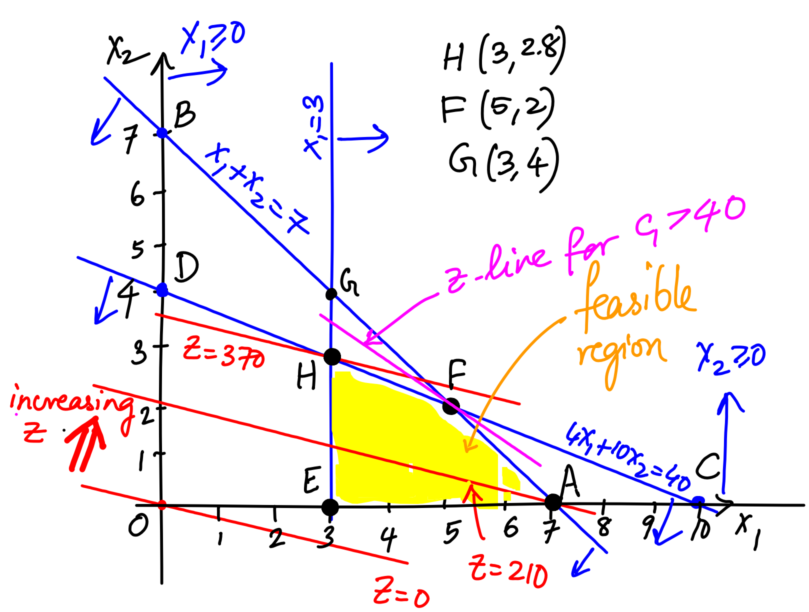
## Recall: Farmer Jones LP:

$$\max \quad z = 30x_1 + 100x_2 \qquad \text{(total revenue)}$$

s.t.

$$x_1 + x_2 \le 7 \qquad \text{(land availability)}$$
$$4x_1 + 10x_2 \le 40 \qquad \text{(labor hrs)}$$
$$10x_1 \ge 30 \qquad \text{(min corn)}$$
$$x_1,\ x_2 \ge 0 \qquad \text{(non-negativity)}$$

H(3,2.8) is the optimal solution.




**Q.** For what values of revenue/acre of corn (currently 30) is the current solution H(3, 2.8) optimal?

# Effect of Change in an objective function Coefficient

Say, price/bu of corn goes up to \$4 (from \$3). Should Jones still farm 3 acres of corn and 2.8 acres of wheat?

So, objective function is $\max z = c_1 x_1 + 100x_2$

More generally, let revenue/acre of corn = \$ $c_1$, for what values of $c_1$ is the current solution optimal?

If objective function is $\max z = c_1 x_1 + 100 x_2$, the slope of z-line is $-\frac{c_1}{100}$.

When $c_1 = 40$, slope of z-line = slope of (labor hrs) line.

For $c_1 \geq 40$, F(5,2) becomes the optimal solution, until $c_1 = 100$. When $c_1 \geq 100$, A(7,0) becomes the optimal solution.

Hence H(3,2.8) is the unique optimal solution for

$\boxed{c_1 \leq 40}$ → assuming $c_1 \geq 0$.

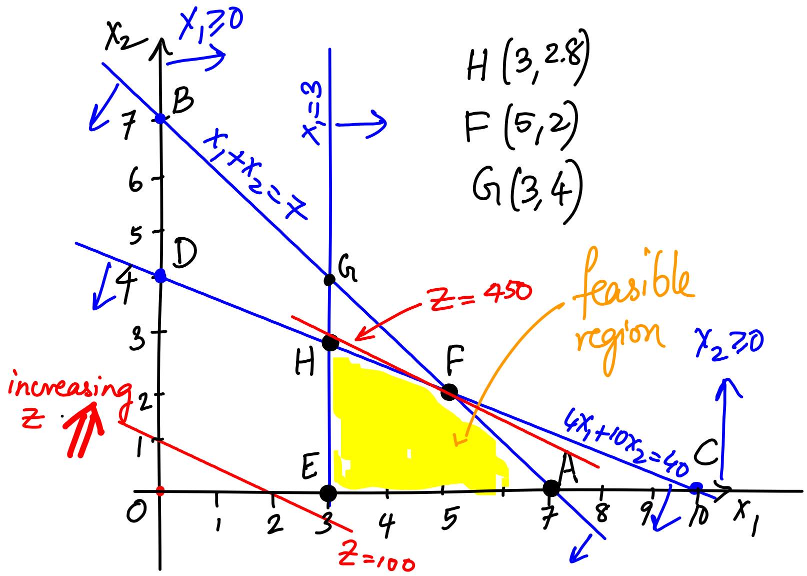
If $c_1$ goes negative, the slope will change sign. But since $c_1$ is the revenue/acre of corn, $c_1 \geq 0$ makes sense.

## Changing revenue/acre of wheat

First, let's assume price/bushel of corn is \$5. So the objective function is max $z = 50x_1 + 100x_2$. Now, $F(5,2)$ is optimal, with $z^* = 450$. The analysis becomes more interesting here, as compared to the original Farmer Jones LP.

$$\begin{array}{llll} \max & z = \cancel{30}\,50x_1 + 100x_2 & & \text{(total revenue)} \\ \text{s.t.} & x_1 + x_2 & \le 7 & \text{(land availability)} \\ & 4x_1 + 10x_2 & \le 40 & \text{(labor hrs)} \\ & 10x_1 & \ge 30 & \text{(min corn)} \\ & x_1,\ x_2 & \ge 0 & \text{(non-negativity)} \end{array}$$

Optimal solution is at $F(5,2)$, with $z^* = 450$.




Q. For what values of revenue/acre of wheat ($c_2$; = 100 now) is the current solution $F(5,2)$ optimal?

We'll finish this topic in the next lecture...