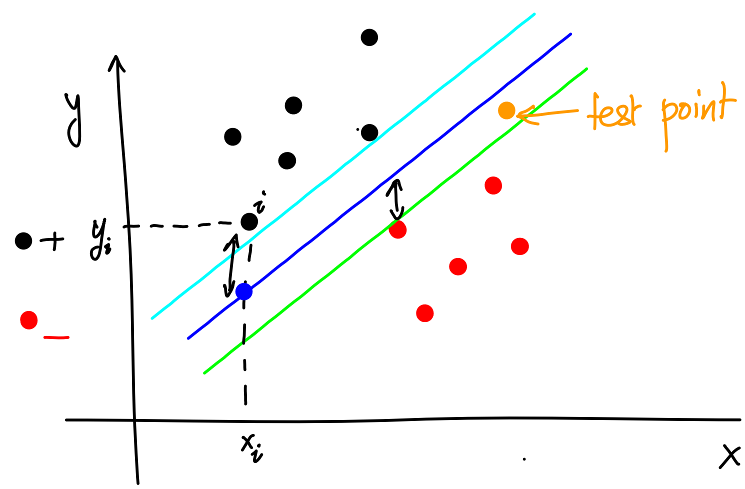
MATH 364: Lecture 25 (11/12/2024)

Today: * about the project
* IP formulations

Project: Motivated by support vector machines (SVM)

In (linear) regression, you fit a straight line that best represents the given set of data points. This line minimizes the sum of squared errors. This is a non-linear objective function.



Here, we want to find a "separating line" which separates the +1 and −1 points as "widely" as possible.

variables

$$y_i = \bar{w}^T \bar{x}^i + w_0 \qquad y_i, \bar{x}^i \text{ are data here}$$

Data has $y_i \in \{+1, -1\}$, so, ideally, we want

$$\bar{w}^T \bar{x}^i + w_0 = +1 \quad \text{for } y_i = +1 \text{ instances.}$$

In fact, we want $\bar{w}^T \bar{x} + w_0 = 1 + \varepsilon_i$, $\varepsilon_i > 0$ if possible.

Solve an LP to identify $(\bar{w}, w_0)$. Then use this predictor to predict on the test set.

There are two LP models suggested in the project description. You may still have to modify the models to get best results. Similarly, you may have to play around with the parameters, e.g., upper bounds on some of the variable values to get the best results.

Getting the data read into AMPL

Since the data is given in text files (TrainingSet.txt and TestSet.txt), one could read in these data files directly into AMPL using the **read** command.

See the course web page for an example.

Integer (Linear) Programming (IP)

An LP in which each variable is restricted to take only integer values is called pure integer program, or integer program (IP) by default.

e.g., $\max\ z = 3x_1 + 2x_2$

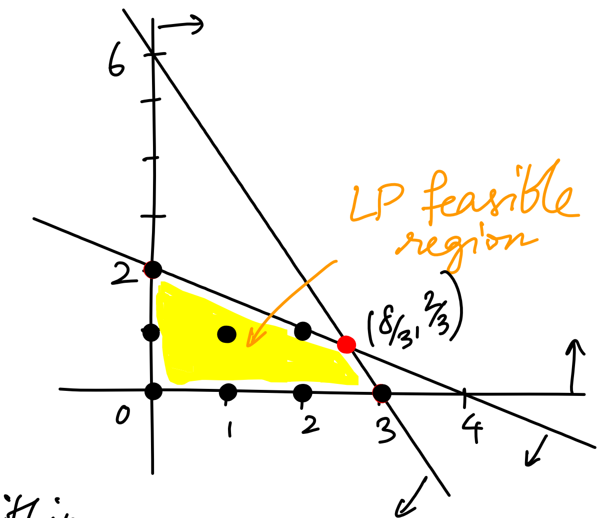
s.t.

$$x_1 + 2x_2 \le 4$$

$$2x_1 + x_2 \le 6$$

$$x_1, x_2 \ge 0,\ x_1, x_2 \text{ integer}$$

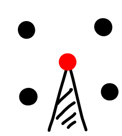
(LP) (IP)



The feasible set here consists of the 8 points with integer coordinates within P, the LP feasible region.

The LP optimal solution is at $(\frac{8}{3}, \frac{2}{3})$, while the IP optimal solution is at $(3, 0)$ ($z^* = 9$ at $(3, 0)$).

Rounding the LP optimal solution works here, but might not work always.



All possible roundings are IP-infeasible here!

An LP in which a subset of variables is restricted to take integer values is called mixed integer program (MIP).

If the integer variables can take values only in $\{0, 1\}$, then the IP is called a Binary IP (BIP).

If we drop the integer restriction from an IP or MIP, we get its LP-relaxation.

IP Formulations

1 Coach Night is trying to choose the starting lineup for the basketball team. The team consists of seven players who have been rated (on a scale of 1 = poor to 3 = excellent) according to their ball-handling, shooting, rebounding, and defensive abilities. The positions that each player is allowed to play and the player's abilities are listed in Table 9.

The five-player starting lineup must satisfy the following restrictions:

1 At least 4 members must be able to play guard, at least 2 members must be able to play forward, and at least 1 member must be able to play center.

2 The average ball-handling, shooting, and rebounding level of the starting lineup must be at least 2.

3 If player 3 starts, then player 6 cannot start. → both 3 & 6 cannot start together

4 If player 1 starts, then players 4 and 5 must both start.

5 Either player 2 or player 3 must start.

Given these constraints, Coach Night wants to maximize the total defensive ability of the starting team. Formulate an IP that will help him choose his starting team.

TABLE 9

Player	Position	Ball-Handling	Shooting	Rebounding	Defense
1	G	3	3	1	3
2	C	2	1	3	2
3	G-F	2	3	2	2
4	F-C	1	3	3	1
5	G-F	3	3	3	3
6	F-C	3	1	2	3
7	G-F	3	2	2	1

Decisions: For each player, do they start or not: YES/NO.

d.V.s : $x_j = \begin{cases} 1 & \text{if player } j \text{ starts} \\ 0 & \text{otherwise} \end{cases}, \quad j = 1, \dots, 7$

Constraints

0. $\sum_{j=1}^{7} x_j = 5$ (five starters)

1. $x_1 + x_3 + x_5 + x_7 \geq 4$ (guards)

$x_3 + x_4 + x_5 + x_6 + x_7 \geq 2$ (forwards)

$x_2 + x_4 + x_6 \geq 1$ (center)

2. $$\frac{3x_1+2x_2+2x_3+x_4+3x_5+3x_6+3x_7}{5 \text{ (or } \sum_j x_j)} \geq 2 \quad \text{(avg ball-handling)}$$

$$3x_1 + \cdots + 2x_7 \geq 5\times 2 \quad \text{(avg. shooting)}$$

$$x_1 + \cdots + 2x_7 \geq 5\times 2 \quad \text{(avg. rebounding)}$$

3. $x_3 + x_6 \leq 1$ (3 and 6 cannot both start)

or $x_6 \leq 1 - x_3$ (if 3 starts, 6 cannot)

$x_3 + x_6 = 1$ insists exactly one of 3 and 6 has to start

4. $x_4 \geq x_1$ (if 1 starts, 4 and 5 must also start)

$x_5 \geq x_1$

One constraint:

$\cancel{x_4 + x_5 \geq x_1 + 1}$? as $x_1 = 0$ would still insist $x_4 + x_5 \geq 1$

But

$x_4 + x_5 \geq 2x_1$ works. $x_1 = 1 \Rightarrow x_4 + x_5 \geq 2$

$x_1 = 0 \Rightarrow x_4 + x_5 \geq 0$, which is redundant

$x_4 = x_1$, $x_5 = x_1$ insists 1, 4, and 5 all start together or all not start.

5. $x_2 + x_3 \geq 1$ (2 or 3 must start, or both)

$x_2 + x_3 = 1$ $\rightarrow$ exactly one of them starts

Objective function:

$$\max \ z = 3x_1 + 2x_2 + \cdots + x_7 \quad \text{(total defense ability)}$$

If we were maximizing the average defense ability, we would set the objective function as

$$\max \ z = \frac{1}{5}(3x_1 + 2x_2 + \cdots + x_7) \quad \text{(avg. defense ability)}$$