MATH 364: Lecture 27 (11/19/2024)

Today: * project submission details
* if-then statements as MIPs

Project * Solve LPs in AMPL;

- LP would give you w_0 and $\bar{w} = [w_1 \dots w_n]^T$ weights.
- you still need to come up with a rule to convert the $y_i = \bar{w}^T \bar{x}_i + w_0$ for the traning set instances to +1/−1. Say, the y_i values lie between −4050 and +8623. Pick a cutoff, say δ, in this range, and make the rule that

  if $y_i \geq \delta$, assign +1
  else assign −1.

  Choice of δ is part of training. Pick δ so that as many (out 90) instances in the training set are assigned correctly.
- Repeat the prediction computations for test set:
  compute $y_i = \bar{w}^T \bar{x}_i + w_0$ for $i = 1, \dots, 10$.
  Then apply the same δ to convert y_i to +1/−1.
- you're welcome to do other calculations outside AMPL (except LP part).

Report (≤ 4 pages)

* (Do **not** include the model file in the report — submit them separately ...)

* Describe which LPs you used.

* Specify ranges of values for parameters you tried. (provide brief justifiction).

* report the accuracy on the test set (how many out of 10 you predicted correctly).

* comment on why you got these results.

Submit all files including report, etc., as a one compressed folder (e.g., zip, ~~tar-gzipped~~, etc.).

→ follow instructions given in the project.

If-Then constraints

Recall: either-or constraints: $f(\cdot) \le 0$ —— (1)
$g(\cdot) \le 0$ —— (2)

Either (1) or (2) should hold:

$$\boxed{\begin{array}{c} f(\cdot) \le My \\ g(\cdot) \le M(1-y) \\ y \in \{0,1\} \end{array}}$$

If-then constraints

$-g(x_1,\dots,x_n) \le 0$

if $f(x_1,\dots,x_n) > 0$ then $g(x_1,\dots,x_n) \ge 0$

If $f(\cdot) > 0$ then we want $g(\cdot) \ge 0$ to hold. But $f(\cdot) > 0$ means $f(\cdot) \le 0$ does **not** hold. We convert the input statement to an equivalent either-or statement.

$A \Rightarrow B$ or (if A then B) $\equiv$ $\underline{\text{not } A}$ or B

"implies" "equivalent to" negation or opposite of A

$\equiv$ either $f(x_1,\dots,x_n) \le 0$ or $-g(x_1,\dots,x_n) \le 0$.

$$\boxed{\begin{array}{c} f(x_1,\dots,x_n) \le My \\ -g(x_1,\dots,x_n) \le M(1-y) \\ y \in \{0,1\} \end{array}}$$

12 A company is considering opening warehouses in four cities: New York, Los Angeles, Chicago, and Atlanta. Each warehouse can ship 100 units per week. The weekly fixed cost of keeping each warehouse open is $400 for New York, $500 for Los Angeles, $300 for Chicago, and $150 for Atlanta. Region 1 of the country requires 80 units per week, region 2 requires 70 units per week, and region 3 requires 40 units per week. The costs (including production and shipping costs) of sending one unit from a plant to a region are shown in Table 11. We want to meet weekly demands at minimum cost, subject to the preceding information and the following restrictions:

1 If the New York warehouse is opened, then the Los Angeles warehouse must be opened.

2 At most two warehouses can be opened.

3 Either the Atlanta or the Los Angeles warehouse must be opened.

Formulate an IP that can be used to minimize the weekly costs of meeting demand.

f_i

$d_j \to$ demands.

TABLE 11

From	To ($) Region 1	Region 2	Region 3
New York	20	40	50
Los Angeles	48	15	26
Chicago	26	35	18
Atlanta	24	50	35

c_{ij}

Decisions

1. Should a warehouse be opened in City i or not, $i = N, L, C, A$ (for NY, LA, Ch, At).

2. How many units to ship from warehouse i to region j, $i = N, L, C, A$, $j = 1, 2, 3$.

d.V.'s

Let $y_i = \begin{cases} 1 & \text{if warehouse opened in city } i \\ 0 & \text{otherwise} \end{cases}$, $i = N, L, C, A$

and x_{ij} = # units shipped from warehouse i to region j, $i = N, L, C, A$, $j = 1, 2, 3$. (≥ 0).

Let f_i = fixed charge at city i ($400 for NY, ...) and

c_{ij} = unit shipping charge from city i to region j (in Table)

d_j = demand in region j. $j = 1, 2, 3$ (80, 70, 40)

for compact representation of MIP model

Objective function

$$\min \quad z = \sum_{i=N,L,C,A} f_i y_i + \sum_{i=N,L,C,A} \sum_{j=1}^{3} c_{ij} x_{ij} \quad \text{(total cost)}$$

Constraints

M_i here

$$\sum_{j=1}^{3} x_{ij} \le 100 y_i \,, \quad i = N, L, C, A \quad \text{(forcing constraints)}$$

Alternatively, we could split up these constraints:

$$x_{ij} \le d_j y_i \,, \quad i = N, L, C, A, \quad j = 1,2,3 \quad \text{(max shipping)}$$

works, as $d_j \le 100$ for each j here

But in this case, we need to write

$$\sum_{j=1}^{3} x_{ij} \le 100 \,, \quad i = N, L, C, A \quad \text{(max shipping)}$$

$$\sum_{i=N,L,C,A} x_{ij} \ge d_j \,, \quad j = 1,2,3 \quad \text{(Region } j \text{ demand)}.$$

Logical constraints

1. If NY then LA $\equiv$ if $\underbrace{y_N > 0}_{\equiv (y_N = 1)}$ then $\underbrace{y_L \ge 1}_{\equiv (y_L = 1)}$ $\equiv \underbrace{y_L - 1}_{g(\cdot)} \ge 0$

$f(\cdot)$

$$y_N \le z_1$$
$$-(y_L - 1) \le 1 - z_1$$
$$z_1 \in \{0, 1\}$$

Used $M = 1$ here; $-g(\cdot) \equiv -(y_L - 1)$
$= 1 - y_L$

if $y_N > 0$ then $y_L \geq 1$ $\equiv$ either $y_N \leq 0$ or $1 - y_L \leq 0$

$$\left.\begin{array}{l} y_N \leq z_1 \\ 1 - y_L \leq 1 - z_1 \end{array}\right\} \text{adding these statements gives } y_N \leq y_L$$

$$z_1 \in \{0,1\}$$

Alternatively, using direct logic, we could write

$y_L \geq y_N$ (if NY then LA)

2. $\sum_{i=N,L,C,A} y_i \leq 2$ (at most 2 warehouses open)

3. Either A or L : $y_A \geq 1$ or $y_L \geq 1$

$\equiv$ $1 - y_A \leq 0$ or $1 - y_L \leq 0$

$$\left.\begin{array}{l} 1 - y_A \leq z_3 \\ 1 - y_L \leq 1 - z_3 \end{array}\right\} y_A + y_L \geq 1$$

$$z_3 \in \{0,1\}$$

Or, using logic, $y_A + y_L \geq 1$

Variable restrictions

$y_i \in \{0,1\}$, $i = N, L, C, A$

$x_{ij} \geq 0$, $i = N, L, C, A$, $j = 1,2,3$

Could write $1 \leq x \leq 4$ x integer

Next lecture: Model: $\underset{3}{x=1}$ or $\underset{8}{x=2}$ or $\underset{13}{x=3}$ or $\underset{21}{x=4}$