# MATH 364: Lecture 29 (12/03/2024)

Today: * Problems from Hw8
* Practice final

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* Final exam will be posted on Wed, Dec 11
* Due by 10 pm on Thu, Dec 12 by email.
* Limited open resource exam!
  ✓ anything posted on course web page
  ✓ Can use AMPL

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## Hint on AMPL implementation for project:

* Declare params in model file for both training and test sets.
* Solve LP on training set data, then use the solution to evaluate on test set data at the ampl: prompt.

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* No need to show any output from AMPL, or any model/data files **in your report PDF**.
  Include all AMPL files in your submission (separate from the report PDF).

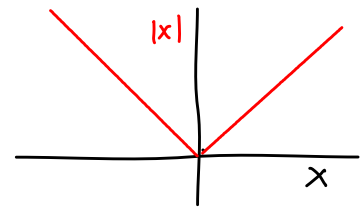
# Problems from Homework

## Hw8. Problem 1

could model as an LP if it's min!

$$\text{min } \cancel{\max}\ z = |3x_2 - 4x_1|$$
$$\text{s.t.} \quad 6x_1 + 2x_2 \le 7$$
$$3x_1 + 4x_2 \le 4$$
$$x_1, x_2 \ge 0$$

min ↓




$|x| = \max\{x, -x\}$.

Recall: $x$ urs $\longrightarrow x^+ - x^-$, $x^+, x^- \ge 0$

So, one could possibly write
$$\max\ z = z^+ + z^-$$
$$\text{s.t.} \quad z^+ - z^- = 3x_2 - 4x_1$$
$$6x_1 + 2x_2 \le 7$$
$$3x_1 + 4x_2 \le 4$$
$$x_1, x_2, z^+, z^- \ge 0$$

But this LP is unbounded.

Say $z = 3x_2 - 4x_1 = \alpha$ is the largest value it can take. Hence

$z^+ = \alpha$, $z^- = 0$ could be a valid solution.

Here, $z = z^+ + z^- = \alpha$, is what you want.

But, $z^+ = 23\alpha$, $z^- = 22\alpha$ gives $z^+ - z^- = \alpha$, while giving you $z^+ + z^- = 45\alpha \gg \alpha$

More generally, max{max} or min{min} cannot be modeled as a linear program. min{max} or max{min} could be modeled.

Here, you have to consider two separate LPs

$$\begin{array}{lll} \max\ z^+ = 3x_2 - 4x_1 & & \min\ z^- = 3x_2 - 4x_1 \\ \text{s.t. } 6x_1 + 2x_2 \le 7 & \text{and} & \text{s.t. } 6x_1 + 2x_2 \le 7 \\ \quad 3x_1 + 4x_2 \le 4 & & \quad 3x_1 + 4x_2 \le 4 \\ \quad x_1, x_2 \geq 0 & & \quad x_1, x_2 \geq 0 \end{array}$$

Then take $\max\{|z^{+*}|, |z^{-*}|\}$, and the corresponding optimal solution $(x_1^*, x_2^*)$ as the answer.

<u>Prob 2 (HW8)</u>

Property holds at start:

$$\begin{array}{cc} x^+ & x^- \\ \hline c & -c \\ \hline a_{11} & -a_{11} \\ \vdots & \vdots \\ \frac{1}{\beta}(a_{i1} & -a_{i1}) \\ \vdots & \vdots \\ a_{m1} & -a_{m1} \\ \hline \end{array}$$

1. Scaling ERO: Divide by $\beta \neq 0$. (typically, $\beta > 0$).

$$\frac{1}{\beta}(a_{i1} \quad -a_{i1}) \rightarrow \frac{a_{i1}}{\beta} \quad -\frac{a_{i1}}{\beta} \checkmark$$

2. Replacement ERO: $R_i \leftarrow R_i + \alpha R_j$

$$a_{i1} \ -a_{i1} \longrightarrow a_{i1} + \alpha a_{j1} \qquad -a_{i1} + \alpha(-a_{j1})$$

$$\longrightarrow (a_{i1} + \alpha a_{j1}) \qquad -(a_{i1} + \alpha a_{j1}) \checkmark$$

$i$ could be 0 here (for Row-0).

# Prob 3, Hw 8

(a) Let $x_j$ replace $x_\ell$, which is currently basic in Row-$i$.

Since $x_j$ is entering (in a max-LP), its coefficient in Row-0 should be $\leq 0$.

$$
\begin{array}{r|cc|l}
 & x_\ell & x_j & \\
\hline
 & 0 & -c_j & c_j > 0 \\
\hline
 & 0 & \vdots & \\
 & \vdots & \vdots & \\
i \rightarrow & 1 & a_{ij} > 0 \text{ (pivot)} & \\
 & 0 & \vdots & \\
 & \vdots & \vdots & \\
 & 0 & & \\
\hline
\end{array}
$$

We do $R_0 + \left(\frac{c_j}{a_{ij}}\right) R_i$ to zero out $-c_j$ (in Row-0) under $x_j$. Under $x_\ell$ in Row-0, we get

$$0 + \left(\frac{c_j}{a_{ij}}\right) 1 = \frac{c_j}{a_{ij}} > 0 \quad \left(\text{as both } c_j > 0 \text{ and } a_{ij} > 0\right).$$

could be $= 0$ if $c_j = 0$.

It is important to detail the effects of EROs in this fashion.

(b) Since coefficient of $x_\ell$ in Row-0 is $> 0$ $(\geq 0)$, it cannot enter back immediately into the basis of a max LP.

# Practice Final Exam

6.

$$(P)\quad \begin{array}{lll} \min & z = 3x_1 + 3x_2 + 4x_3 & \\ \text{s.t.} & 4x_1 + 6x_2 + 3x_3 & \geq 7 \quad y_1 \geq 0 \\ & 3x_1 + x_2 + x_3 & \geq 3 \quad y_2 \geq 0 \\ & x_1,\ x_2,\ x_3 & \geq 0 \\ & \leq \quad \leq \quad \leq & \end{array}$$

$$(D)\quad \begin{array}{lll} \max & w = 7y_1 + 3y_2 & \\ \text{s.t.} & 4y_1 + 3y_2 \leq 3 & s_1 \\ & 6y_1 + y_2 \leq 3 & s_2 \\ & 3y_1 + y_2 \leq 4 & s_3 \geq 0 \\ & y_1, y_2 \geq 0 & \end{array}$$

As $s_3 = \frac{16}{7}$, $x_3 = 0$ (CSC).

From AMPL: $y_1 = \frac{3}{7}$, $y_2 = \frac{3}{7}$, $w^* = \frac{30}{7}$.

It would be efficient to use AMPL to solve (D) here. At the same time, you could verify the optimal solution for (P) as well!

$$3y_1 + y_2 = 3\left(\frac{3}{7}\right) + \left(\frac{3}{7}\right) = \frac{12}{7} = 4 - \frac{16}{7}. \text{ So } s_3 = \frac{16}{7}.$$

Since $s_3 > 0$, CSCs give $x_3 = 0$. (as $s_3 x_3 = 0$).

Also, since $y_1 > 0$ and $y_2 > 0$, CSCs give $s_1 = s_2 = 0$ ($s_i y_i = 0$).

Hence in (P), we have

$$\begin{aligned} 4x_1 + 6x_2 &= 7 \\ 3x_1 + x_2 &= 3 \\ \hline \Rightarrow \quad 14x_1 &= 11 \end{aligned} \quad \Rightarrow x_1 = \frac{11}{14}, x_2 = \frac{9}{14}.$$

Indeed, $z^* = 3x_1 + 3x_2 = 3\left(\frac{11+9}{14}\right) = \frac{30}{7} = w^*$, as expected.

AMPL model of (D)

```
var y1 >= 0;
var y2 >= 0;

maximize w: 7*y1 + 3*y2;

s.t.  x1:       4*y1 + 3*y2 <= 3;
s.t.  x2:       6*y1 +   y2 <= 3;
s.t.  x3:       3*y1 +   y2 <= 4;
```

AMPL session:

```
ampl: reset; model Pr6_PracFinal.txt; solve; display y1,y2;
Gurobi 10.0.0: optimal solution; objective 4.285714286
2 simplex iterations
y1 = 0.428571
y2 = 0.428571

ampl: display x1,x2,x3;
x1 = 0.785714
x2 = 0.642857
x3 = 0
```

$4.285714286 \to \frac{30}{7}$, $y_1, y_2 \to 3/7$, $x_1 \to 11/14$, $x_2 \to 9/14$