MATH 364: Lecture 30 (12/05/2024)

Today: Practice final exam.

---

3. Word selection IP:

Let $x_j = 1$ if word j is selected, and 0 otherwise, (0 or 1).

$j=1 \equiv$ AFT, $j=2 \equiv$ FAR, ..., $j=7 \equiv$ ZAP.

Let l_i = sum of letter i scores, $i=1,2,3$. (≥ 0)

Data: s_i = total score for word i.

s_1 = score(AFT) = 27, ..., s_7 = score(ZAP) = 43.

$$\max\ z = \sum_{i=1}^{7} s_i x_i \qquad \text{(total score)}$$

s.t.

$$l_1 = \underset{A}{x_1} + \underset{F}{6x_2} + \cdots + \underset{Z}{26x_7} \qquad \text{(letter 1 score)}$$

$$l_2 = \underset{F}{6x_1} + \underset{A}{x_2} + \cdots + \underset{A}{x_7} \qquad \text{(letter 2 score)}$$

$$l_3 = \underset{T}{20x_1} + \underset{R}{18x_2} + \cdots + \underset{P}{16x_7} \qquad \text{(letter 3 score)}$$

$$\sum_{i=1}^{7} x_i = 4 \qquad \text{(pick 4 words)}$$

$$x_2 \leq 1 - x_7 \qquad \text{(ZAP} \Rightarrow \text{no FAR)}$$

$$x_3 = x_4 \qquad \text{(JOE \& KEN, or neither)}$$

l_1, l_2, l_3 will all be integers. We want $l_1 < l_2 < l_3$.
Hence we can write

$l_1 \le l_2 - 1$ (letter 1 score < let. 2 score)

$l_2 \le l_3 - 1$ (letter 2 score < let. 3 score)

$x_j \in \{0,1\}, \ j = 1, \ldots, 7$ (Binary vars)

5. **if** $|2x + 5y| > 2$ **then** $|3x + 4y| \ge 5$.

if $|x| \le 2$ then $-2 \le x \le 2$.

if $|x| \ge 3$ then $x \ge 3$ or $x \le -3$

$\equiv$ either $|2x+5y| \le 2$ or $|3x+4y| \ge 5$

$\equiv$ either $(2x+5y \le 2$ AND $2x+5y \ge -2)$ OR
$(3x+4y \ge 5$ OR $3x+4y \le -5)$

$\equiv$ either $(\underset{(1)}{2x+5y-2 \le 0}$ AND $\underset{(2)}{-2x-5y-2 \le 0})$ OR
$(\underset{(3)}{-3x-4y+5 \le 0}$ OR $\underset{(4)}{3x+4y+5 \le 0})$

Let $t_i = 1$ if statement (i) holds; $i = 1,2,3,4$.
But (1) AND (2) is one option, so we use t_1 in place of t_2.

$$\left.\begin{aligned} 2x+5y-2 &\le M(1-t_1) \\ -2x-5y-2 &\le M(1-t_1) \end{aligned}\right\}$$

OR, you could use t_2 for (2), but write $t_1+t_2+t_3+t_4 \ge 1$ & $t_1 = t_2$.

$$-3x-4y+5 \le M(1-t_3)$$
$$3x+4y+5 \le M(1-t_4)$$
$$t_1 + t_3 + t_4 \ge 1$$
$$t_i \in \{0,1\}, \ i = 1,3,4$$