MATH 364: Lecture 7 (09/10/2024)

Today: * one more formulation problem
* AMPL

We first finish the inventory planning LP...

d.v.'s

Let x_i = # units produced in month i, $i=1,\dots 4$

s_i = # units stored from month i to $i+1$, $i=0,\dots,4$

s_0 = starting inventory $(=0)$.

The problem did not mention anything about units available at start of month 1. We capture this quantity in s_0 — and can write all flow-balance constraints in a unified manner using s_0 (and s_i, x_i for $i=1$–4).

Objective function

$$\min \quad z = \underbrace{5x_1 + 8x_2 + 4x_3 + 7x_4}_{\text{prodn cost}} + \underbrace{2s_1 + 2s_2 + 2s_3}_{\text{storage cost}} - 6s_4 \qquad \text{(total cost)}$$

revenue at end

Constraints

$s_0 + x_1 = 50 + s_1$ (inventory balance month 1)

$s_1 + x_2 = 65 + s_2$ (inv. balance month 2)

$s_2 + x_3 = 100 + s_3$ (inv. balance month 3)

$s_3 + x_4 = 70 + s_4$ (inv. balance month 4)

$s_0 = 0$ (no starting inventory)

$s_i, x_i \geq 0$, for all i (non-neg).

If we let d_i be the demand in month i (this is data given to us), we could write the balance constraints for all months in one go:

$$\underbrace{s_{i-1} + x_i}_{\text{inflow}} = \underbrace{d_i + s_i}_{\text{outflow}}, \quad i = 1\text{–}4 \qquad \text{(inv. balance month } i\text{)}$$

Here, $d_1 = 50$, $d_2 = 65$, etc.

Also note that we do not have to write additional constraints that ensure all demand is met. We would write

$$\underbrace{s_{i-1} + x_i - s_i}_{\text{net inflow}} \geq d_i \qquad \text{(meet demand month } i\text{)}$$

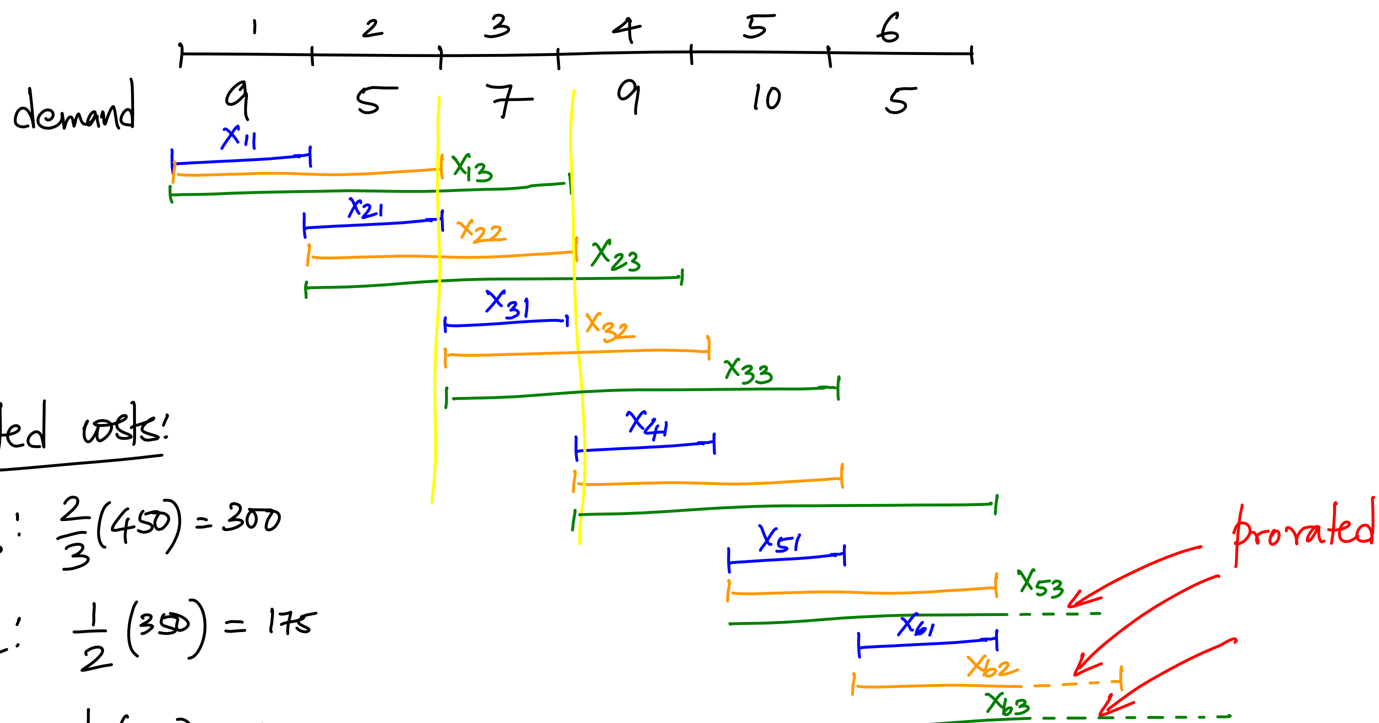
But this constraint is forced (at equality) by the balance constraints. And we do need the balance constraints, so we could skip the demand constraints once the flow balance constraints are written.

We consider a final formulation LP that is similar in flavor to the Small Town Police scheduling LP

2 An insurance company believes that it will require the following numbers of personal computers during the next six months: January, 9; February, 5; March, 7; April, 9; May, 10; June, 5. Computers can be rented for a period of one, two, or three months at the following unit rates: one-month rate, \$200; two-month rate, \$350; three-month rate, \$450. Formulate an LP that can be used to minimize the cost of renting the required computers. You may assume that if a machine is rented for a period of time extending beyond June, the cost of the rental should be prorated. For example, if a computer is rented for three months at the beginning of May, then a rental fee of $\frac{2}{3}(450) = \$300$, not \$450, should be assessed in the objective function.

d.v.'s

Let x_{ij} = # computers rented starting in month i on a j-month lease, $i=1,\ldots,6$, $j=1,2,3$.



Prorated costs!

x_{53}: $\frac{2}{3}(450) = 300$

x_{62}: $\frac{1}{2}(350) = 175$

x_{63}: $\frac{1}{3}(450) = 150$

Allowed to prorate only @ end!

Hint on Problem 1 (HW3):

x_{13} = # officers working shifts 1 and 3; similarly, $x_{14}, x_{15}, x_{16}, \quad x_{24}, \ldots, x_{46}$.

In general: $x_{i,i+1}$ and x_{ij} for $j > i+1$
= # officers working shifts $i, i+1$ or i and j.

Here is the computer leasing problem:

$$\min\ z = 200\Big(\sum_{i=1}^{6} \underbrace{x_{i1}}_{\text{1-month leases}}\Big) + 350\Big(\sum_{i=1}^{5} \underbrace{x_{i2}}_{\text{2-month leases}}\Big) + 450\Big(\sum_{i=1}^{4} \underbrace{x_{i3}}_{\text{3-month leases}}\Big) +$$

$$\frac{2}{3}(450)\, x_{53} + \frac{1}{2}(350)\, x_{62} + \frac{1}{3}(450)\, x_{63} \qquad \text{(total cost)}$$

s.t.

$$\begin{array}{rcll}
x_{11} + x_{12} + x_{13} & \geq & 9 & \text{(Jan demand)} \\
x_{12} + x_{13} + x_{21} + x_{22} + x_{23} & \geq & 5 & \text{(Feb demand} \\
x_{13} + x_{22} + x_{23} + x_{31} + x_{32} + x_{33} & \geq & 7 & \text{(Mar demand)} \\
x_{23} + x_{32} + x_{33} + x_{41} + x_{42} + x_{43} & \geq & 9 & \text{(Apr demand)} \\
x_{33} + x_{42} + x_{43} + x_{51} + x_{52} + x_{53} & \geq & 10 & \text{(May demand)} \\
x_{43} + x_{52} + x_{53} + x_{61} + x_{62} + x_{63} & \geq & 5 & \text{(Jun demand)} \\
\text{all } x_{ij} & \geq & 0 & \text{(non-neg)}
\end{array}$$

See the AMPL handout, AMPL session, and the lecture video ...