Linear Optimization (Spring 2023): Homework 1

- The total points (given in parentheses) add up to 80.
- **You must email your submission as a PDF file to kbala@wsu.edu**. You are welcome to write answers by hand, and scan the writings **into a PDF file**.
- **Your file name should identify you in the following manner. If you are Tweek Tweak, you should name your submission TweekTweak_Hw1.pdf. If you want to add more bits to the title, e.g., Math464, you could name it TweekTweak_Math464_Hw1.pdf, for instance. But you should start the file name with TweekTweak. And it is NOT "Tweek Tweak" or "Tweek_Tweak" or ...**
- **Begin the SUBJECT of your email submission with the same `FirstnameLastname`, e.g., "TweekTweak Hw1 submission".**
- **This homework is due by 4:59 PM on Thursday, January 19**.

0. (15) Meet with me briefly (on Zoom or in person). Do so even if you've taken a class from me in the past.

1. (15) Consider the following system of linear equations.

$$\begin{aligned} x_1 + 3x_2 &= k \\ x_1 - hx_2 &= 2 \end{aligned}$$

Determine all the values of the parameters h and k for which each of the following statements are true.

   (a) The system has no solution.

   (b) The system has a unique solution.

   (c) The system has many solutions.

2. (10) Let λ be an eigenvalue of the $n \times n$ matrix A. Let $B = A - \lambda I$. Show that B is not an invertible matrix.

3. (25) Let $T : V \to W$ be a linear transformation from vector space V to vector space W, both of which are finite-dimensional. Let H be a nonzero subspace of V, and let $T(H) = \{T(\mathbf{x}) \,|\, \mathbf{x} \in H\}$ (i.e., $T(H)$ is the set of images of the vectors in H). Prove that $T(H)$ is a subspace of W. Also show that $\dim(T(H)) \leq \dim(H)$. Recall that $\dim(H)$ denotes the dimension of the vector space H.

4. (15) Let the matrix $A = \begin{bmatrix} B & O \\ O & C \end{bmatrix}$, where B, C are square matrices, and O is a zero matrix. Show that A is invertible if and only if both B and C are invertible.