Linear Optimization (Spring 2023): Homework 5

- The total points (given in parentheses) add up to 140. You will be graded for 130 points (with the possibility of getting up to 10 points as extra credit).
- BT-ILO stands for the text (Bertsimas and Tsitsiklis: Introduction to Linear Optimization).
- **You must email your submission as a PDF file to kbala@wsu.edu**. You are welcome to write answers by hand, and scan the writings **into a PDF file**.
- **Your file name should identify you in the following manner. If you are Tolkien Black, you should name your submission TolkienBlack_Hw5.pdf. If you want to add more bits to the title, e.g., Math464, you could name it TolkienBlack_Math464_Hw5.pdf, for instance. But you should start the file name with TolkienBlack; and NOT "Token Black" or "Tolkien_Black" or ...**
- **Begin the SUBJECT of your email submission with the same `FirstnameLastname`, e.g., "Tolkien-Black Hw5 submission".**
- **This homework is due by 4:59 PM on Thursday, February 16**.

1. (25) BT-ILO Problem 2.3 from Page 76. You need **not** prove the theorem analogous to Theorem 2.4; just provide the analogous procedure for constructing basic solutions.

2. Consider the following LP:

$$\begin{array}{lrcl} \max & z = 3x_1 + 4x_2 & & \\ \text{s.t.} & 2x_1 + 5x_2 & \leq & 20 \\ & 4x_1 + 3x_2 & \leq & 24 \\ & x_1 + x_2 & \geq & 2 \\ & x_1, \quad x_2 & \geq & 0 \end{array}$$

   (a) (15) Solve the LP graphically. Also identify **ALL** corner points of the feasible region (i.e., give the x_1 and x_2 coordinates for each of them). Name the corner points by letters, e.g., A,B,C, etc.

   (b) (30) Write the LP in standard form. How many basic solutions are there for this LP in standard form? Find the basic solutions corresponding to each corner point identified in part (a). You must clearly indicate the basis (i.e., the column indices that determine the basis; you could use the notation $B(1), B(2), \ldots$), and **show** that the basis is indeed a basis in each case (you could show that the basis matrix B is invertible, or $\det(B) \neq 0$). Then you should find the solution to $B\mathbf{x}_B = \mathbf{b}$. You can use Matlab to solve each of these systems of linear equations. Finally, you must point out the values of x_1 and x_2 for each basic solution, showing the correspondence with the corner points identified in part (a).

   (c) (10) Now add the fourth constraint $14x_1 + 7x_2 \leq 76$. Is there a degenerate bfs now? If yes, identify the corner point on the graph of the feasible region. Then find the basic solution corresponding to this corner point. Demonstrate that there are more than $n - m$ x_j's set at zero at this basic solution.

3. (40) BT-ILO Problem 2.10 from page 77.

4. (20) BT-ILO Problem 2.12 from page 77.