Linear Optimization (Spring 2023): Project

- **You must email your submission as a folder to kbala@wsu.edu**. Include the **report as a PDF file** as well as all **Matlab files** in this folder.
- **Your folder name should identify you in the following manner. If you are Costof Doing Business, you should name your submission folder CostofBusiness_Project. If you want to add more bits to the title, e.g., Math464, you could name it CostofBusiness_Math464_Project, for instance. Start the folder name with CostofBusiness; and NOT "Costof Business" or "Cost_Business"...**
- Name your **PDF file** as **CostofBusiness_Project.pdf**.
- **Begin the SUBJECT of your email submission with the same `FirstnameLastname`, e.g., "Costof-Business Project submission".**
- **This project is due by 11:59 PM on Tuesday, May 2**.

Introduction

This project involves the implementation of the revised simplex method and the tableau simplex method in Matlab (preferred; but you could try another program), and comparing the performances of your programs with that of standard optimization software (Matlab's own LP solver). You should test these programs on a set of LP problems for which you need to generate the data randomly. You will also submit a concise yet well structured report.

1 Simplex implementations in Matlab

1. (30) Write a Matlab function that solves a linear program in **standard form** using the revised simplex method. The function should take as input the constraint matrix A, the right hand-side vector $\mathbf{b}$, the cost vector $\mathbf{c}$, and output an optimal solution vector $\mathbf{x}$ and the optimal cost, or indicate that the LP is unbounded or infeasible. It should also output the number of simplex pivots, i.e., iterations, used.

   The function should have the flexibility in terms of the rules used for choosing the entering and leaving variables. As far as choosing the entering variable is concerned, the function should provide the choice to implement the following options.

   (a) After calculating all reduced costs, choose the variable with the most negative reduced cost to enter the basis. This should be the default option.
   (b) Calculate the reduced costs **one at a time**, and choose the variable that *first* gives a negative reduced cost to enter. In this option, you must **not** calculate all reduced costs.

   For choosing the leaving variable, the function should provide the following options.

   (a) *Break ties arbitrarily*: Choose any one of the variables giving the min-ratio to leave. This should be the default option.
   (b) *Smallest index rule*: From among the candidates tied for the leaving variable, the variable x_ℓ with the smallest ℓ leaves.

2. (30) Write a Matlab function that solves a linear program in **standard form** using the tableau simplex method. The function should take as input the constraint matrix A, the right hand-side vector $\mathbf{b}$, the cost vector $\mathbf{c}$, and output an optimal solution vector $\mathbf{x}$ and the optimal cost, or indicate that the LP is unbounded or infeasible. It should also output the number of simplex pivots, i.e., iterations, used.

   The function should have the flexibility in terms of the rules used for choosing the entering and leaving variables. As far as choosing the entering variable is concerned, the function should provide the choice to implement the following options.

   (a) After calculating all reduced costs, choose the variable with the most negative reduced cost to enter the basis. This should be the default option.
   (b) After calculating all reduced costs, choose the variable with the smallest index with a negative reduced cost to enter the basis.

   For choosing the leaving variable, the function should provide the following options.

   (a) *Break ties arbitrarily*: Choose any one of the variables giving the min-ratio to leave. This should be the default option.
   (b) *Lexicographic rule*: From among the candidate rows having positive entries in the pivot column, the leaving variable corresponds to the lexicographically smallest row, after scaling by the pivot column entry.

2 An LP problem for comparison

3. (5) Jimbo Enterprises produces n products. Each product can be produced in one of m machines. Let t_{ij} be the time in hours needed to produce one unit of product i on machine j. For month k, the number of hours available on machine j is h_{kj}. Customers are willing to buy **up to** d_{ik} units of product i in month k at the unit cost of c_{ik}. Formulate an LP that Jimbo can use to maximize the revenue by selling the products for the next p months.

2.1 Comparison with `linprog` in Matlab

Linprog is Matlab's LP solver. The latest version of Matlab should come with linprog (type `help linprog` at the Matlab command window to learn more).

4. (30) Generate a reasonably large—at least 100, but more would be better—different instances of the above LP. You can select the parameters $m, n, p, c_{ik}, d_{ik}, h_{kj}$ and t_{ij} randomly. Of course, to get meaningful comparison, you must vary the size parameters m, n, and p over reasonably wide ranges. For each instance, solve the LP using your revised simplex method function, tableau simplex method function, and using the function `linprog` in Matlab. Compare the *running times* and *number of iterations* used by each function as the *size* of the problem increases. For the revised simplex method, you should compare the statistics for both choices of entering variable selection. The leaving variable selection could be set as the default (break ties arbitrarily) in both the revised and the tableau simplex methods. You must describe how you generate the data for your instances in your report (see Section 3).

5. (15) Describe how you would change the description of the Jimbo Enterprises problem and generate instances, i.e., how you would set the data parameters, of the modified LP that are **infeasible**. Comparing the running times and numbers of iterations for detecting infeasibility on these instances using your implementations of revised and tableau simplex methods and `linprog`. Options for choosing entering/leaving variables could be set as the default ones.

3 Project Report and Submission

6. (50) You must submit a concise project report describing the problem formulation, the strategy used to generate the LP instances for both Problems 4 and 5 (feasible and infeasible), and the details of comparisons of running times and numbers of iterations in each case. The maximum length is *five* pages that are single spaced, single column, and with at least 1-inch margins. You could add a clearly marked Appendix in which additional details are provided (the Appendix does not contribute to the five-page limit). But the main report (of five pages) should be self contained, and **you will be graded for the project report based on the main report alone**.

   To improve readability of your report, you should **not** include your code in there. Instead, you should submit the programs separately. Similarly, you could present the running time comparisons compactly in the form of graphs.

You need not submit the complete data for all the instances—one or two sample instances would do, if any are included.