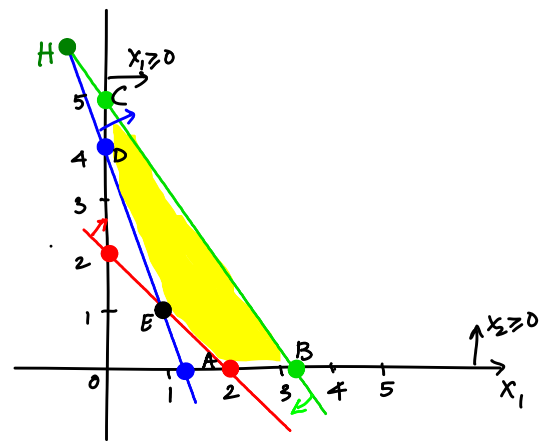
MATH464 – Lecture 18 (03/09/2023)

Today: * Examples of tableau simplex
* cycling in tableau simplex

Back to tableau simplex example on our popular LP:

$$\begin{array}{llr} \min & 2x_1+x_2 & \\ \text{s.t.} & x_1+x_2-x_3 & =2 \\ & 3x_1+x_2 \quad -x_4 & =4 \\ & 3x_1+2x_2 \quad +x_5 & =10 \\ & x_j \geq 0 \ \forall j & \end{array}$$



We start at the bfs $= B(10/3, 0)$, i.e., $\mathcal{B} = \{1,3,4\}$.

We move from B to A to E.

We had done the first iteration (B→A) in the last lecture.
Today, we do the next iteration (A→E).

See the MATLAB session at

https://www.math.wsu.edu/faculty/bkrishna/FilesMath464/S23/Software/Lec18_03092023_Session.txt

Simplex Method and Cycling

BT-ILO Example 3.6 (pg 104)

The book has 3 in place of 0; the value does not change at all, and we get back to this tableau in 6 iterations.

	rhs	x_1	x_2	x_3	x_4	x_5	x_6	x_7
	0	$-3/4$	20	$-1/2$	6	0	0	0
$x_5 =$	0	$1/4$	-8	-1	9	1	0	0
$x_6 =$	0	$1/2$	-12	$-1/2$	3	0	1	0
$x_7 =$	1	0	0	1	6	0	0	1

Notice that the bfs is degenerate here — $x_5 = x_6 = 0$ in the bfs!

Pivoting Rules

1. Choose non-basic x_j with most negative c_j' to enter.
2. If tied for leaving variable, choose one with the smallest index.

   e.g., x_2, x_5 are candidates (in Rows 5 & 3, respectively) pick x_2 to leave, as $2 < 5$ (even though it is basic in a row higher up than the row x_5 is basic in).

We get back to the starting tableau after 6 iterations!

See the MATLAB session at

https://www.math.wsu.edu/faculty/bkrishna/FilesMath464/S23/Software/Lec18_03092023_Session.txt

Ways to avoid cycling

1. lexicographic pivoting rule (dictionary order to choose leaving variable).
2. Bland's rule (min index rule).

Lexicographic ordering (dictionary ordering)

Def Let $\bar{u}, \bar{v} \in \mathbb{R}^n$. $\bar{u}$ is **lexicographically larger** (smaller) than $\bar{v}$ if

1. $\bar{u} \neq \bar{v}$, and
2. the first non-zero component of $\bar{u}-\bar{v}$ is positive (negative).

We write $\bar{u} \overset{L}{\succ} \bar{v}$ ($\bar{u} \overset{L}{\prec} \bar{v}$).

Example

$$\overset{\bar{u}}{(1,2,3,4)} \overset{L}{\succ} \overset{\bar{v}}{(1,2,0,7)} \qquad \bar{u}-\bar{v} = (0,0,3,-3)$$

$$(0,5,8,100) \overset{L}{\prec} (1,0,0,0)$$

When $\bar{u} \overset{L}{\succ} \bar{0}$, we say $\bar{u}$ is lexicographically positive (or lex-positive).

e.g., $\bar{u} = (0,0,1,-5,-7,2)$ is lex-positive.

Notice that $\bar{u}$ need not be positive to be lex-positive!

In the lexicographic pivoting rule, we will scale all candidate rows (for leaving variable) so that the entries in the pivot column are all 1's. Then we pick the lexicographically smallest one as ℓ (so $x_{B(\ell)}$ leaves the basis). More on this approach after the break ...