MATH 464 – Lecture 28 (04/20/2023)

Today: Project

Simplex Implementations

Q. What about starting bfs?

Add an artificial variable per row, and add $+M$ to the cost vector. Choose all artificial varibles in your starting bfs.

Default format of functions desired:

$[\text{status}, z^*, \bar{x}^*, \text{nItr}] = \text{TableauSimplex}(A, \bar{b}, \bar{c}, \text{EntRule}, \text{LvgRule})$

status: optimal/unbounded/infeasible

nItr: # iterations

EntRule: option for entering variable

LvgRule: option for leaving variable

These options could be set as integers, e.g., 1, 2, …, and procedures chosen within your program based on them.

Running Time

Could use tic+toc or use clock() as shown below.

```
t1=tic;
RevisedSimplex;
RunTime = toc(t1);
```

OR

```
t1 = clock();
TableauSimplex;
t2 = clock();
Elapsed_Time = etime(t2,t1);
```

28.2

3. (5) Jimbo Enterprises produces n products. Each product can be produced in one of m machines. Let t_{ij} be the time in hours needed to produce one unit of product i on machine j. For month k, the number of hours available on machine j is h_{kj}. Customers are willing to buy **up to** d_{ik} units of product i in month k at the unit cost of c_{ik}. Formulate an LP that Jimbo can use to maximize the revenue by selling the products for the next p months.

Let x_{ijk} = # units of product i made on machine j in month k (and sold)

$$\max \quad z = \sum_{i=1}^{n} \sum_{k=1}^{p} c_{ik} \left(\sum_{j=1}^{m} x_{ijk} \right) \qquad \text{(total revenue)}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} t_{ij} x_{ijk} \le h_{kj}, \quad j=1,\dots,m, \quad k=1,\dots,p \qquad \text{(max hrs)}$$

$$\sum_{j=1}^{m} x_{ijk} \le d_{ik}, \quad i=1,\dots,n, \quad k=1,\dots,p \qquad \text{(max prods)}.$$

$$x_{ijk} \ge 0 \quad \forall i,j,k \qquad \text{(non-neg)}$$

We need to convert this LP to $A\bar{x} \le \bar{b}$ form (first).

What do $A, \bar{b}, \bar{c}$ look like?

Q: Before that... when will this LP be infeasible?

If h_{kj}, d_{ik}, t_{ij} are all ≥ 0, the LP is guaranteed to be feasible, as $\bar{x} = \bar{0}$ is feasible.

1. You could get an infeasible instance if some of h_{kj}, d_{ik}, t_{ij} are chosen as < 0. OR
2. Change the LP so that d_{ik}'s are demands and you're minimizing total cost (c_{ik}'s can be taken as unit costs). Then, if some d_{ik} are too large for chosen h_{kj} values, the LP could be infeasible.

Back to formulation

Let $n=2, m=2, p=3$.

$$\bar{x}^T = [x_{111}, x_{112}, x_{113}, x_{121}, x_{122}, x_{123}, x_{211}, x_{212}, x_{213}, x_{221}, x_{222}, x_{223}]$$

$$\bar{c}^T = [\underbrace{c_{11}, c_{12}, c_{13}, c_{11}, c_{12}, c_{13}}_{m \text{ times}}, c_{21}, c_{22}, c_{23}, c_{21}, c_{22}, c_{23}]$$

Data C is an $\underline{n \times p}$ matrix. Here $C \in \mathbb{R}^{2\times 3}$: $C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$

Check out: **repmat** and **reshape** in Matlab. $C = \begin{bmatrix} 3 & 4 & 1 \\ 2 & 5 & 6 \end{bmatrix}$

$$\bar{c}^T = [3\ 4\ 1\ 3\ 4\ 1\ 2\ 5\ 6\ 2\ 5\ 6];$$

$$\bar{x}^T = [x_{111}, x_{112}, x_{113}, x_{121}, x_{122}, x_{123}, x_{211}, x_{212}, x_{213}, x_{221}, x_{222}, x_{223}]$$

$$\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \end{array}
\underbrace{\begin{bmatrix}
t_{11} & & & & & & t_{21} & & & & & \\
& t_{11} & & & & & & t_{21} & & & & \\
& & t_{11} & & & & & & t_{21} & & & \\
& & & t_{12} & & & & & & t_{22} & & \\
& & & & t_{12} & & & & & & t_{22} & \\
& & & & & t_{12} & & & & & & t_{22} \\
1 & & & 1 & & & & & & & & \\
& 1 & & & 1 & & & & & & & \\
& & 1 & & & 1 & & & & & & \\
& & & & & & 1 & & & 1 & & \\
& & & & & & & 1 & & & 1 & \\
& & & & & & & & 1 & & & 1
\end{bmatrix}}_{A}
\bar{x} \leq
\overset{\bar{b}}{\begin{bmatrix} h_{11} \\ h_{21} \\ h_{31} \\ h_{12} \\ h_{22} \\ h_{32} \\ d_{11} \\ d_{12} \\ d_{13} \\ d_{21} \\ d_{22} \\ d_{23} \end{bmatrix}}$$

Generating Data

Decide ranges of values for n, m, p. Start small, and increase in steps. First ensure your simplex functions work correctly.

For each given n, m, p, generate data (C, D, T, H) randomly.

You would want to go to large enough values so that you (start to) observe your revised simplex starting to gain on your tableau simplex.