MATH 464 — Lecture 29 (04/25/2023)

Today:
* on the project
* Interior point methods
* Instances "bad" for simplex

More on the Project

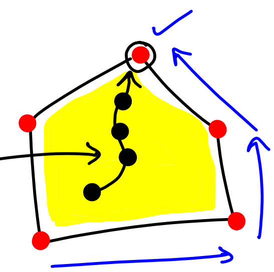
* On input (data) format:
  - ✗ Should not prompt user to input data (entry by entry).
  - ✓ Should work just like linprog().
* You should have a way to either regenerate the same random instances — to solve again, or to solve using different approaches.
  Can either set the seed for random number generators in Matlab deterministically (i.e., record the seed)
  Or you could save the instances themselves.
* Starting bfs, slack variables, and more...
  * Your simplex functions are supposed to solve standard form LPs: $\min\{\bar{c}^T\bar{x} \mid A\bar{x}=\bar{b},\ \bar{x}\geq\bar{0}\}$.
  * If slack variables need to be added to get all constraints to equations, do so before/in the process of creating your A matrix.
  * May be simplest to add artificial variables with big-M to get starting bfs — works for all instances!

Do the project in MATLAB, especially if you have to ask me about doing it in another package/language (meaning you're not confident enough)!

Interior Point Methods

Simplex method $\longrightarrow$ starts at a bfs, moves to adjacent bfs so that cost improves; repeat till termination.

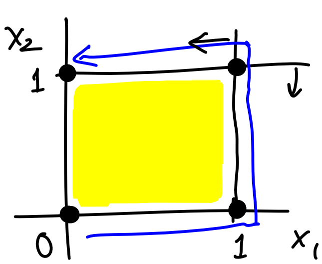
Moves along the boundary of the polyhedron.



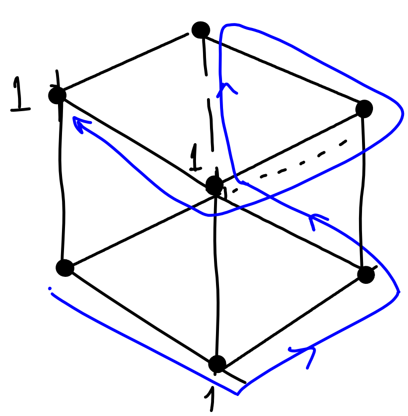
Interior point method $\longrightarrow$ starts at a feasible solution (not necessarily a bfs), move inside the feasible region to improve the cost, till you terminate.

Instances that are "bad" for simplex method

A cube in $\mathbb{R}^n$: $0 \le x_j \le 1 \;\forall j$ $\quad 2^n$ vertices.



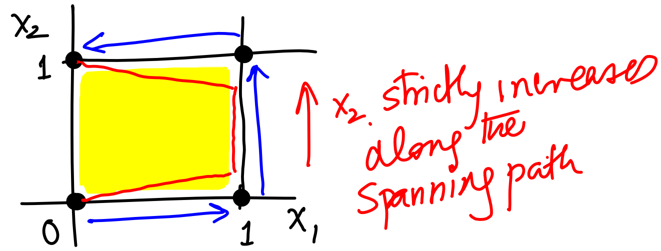
Spanning path: A path traveling along edges, visits every vertex.



Path visiting all 8 vertices of the cube.

If each vertex in this path improves the cost, then the simplex method visits 2^n vertices before it finishes!

We modify the cube a little bit to get this setting:



Let $\epsilon \in (0, 0.5)$. Consider the following set of constraints.

$$\epsilon \le x_1 \le 1$$

$$\epsilon x_{i-1} \le x_i \le 1 - \epsilon x_{i-1}, \quad i = 2, \ldots, n.$$

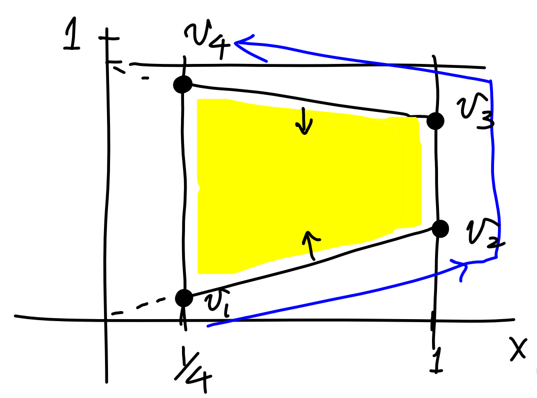
Here is the instance for $\epsilon = \frac{1}{4}$, $n = 2$.

$$\frac{1}{4} \le x_1 \le 1$$

$$\frac{1}{4} x_1 \le x_2 \le 1 - \frac{1}{4} x_1$$

$$x_1 - 4x_2 \le 0$$

$$x_1 + 4x_2 \le 4$$



x_2 increases strictly as we go $v_1 \to v_2 \to v_3 \to v_4$

In the general instance, ~~if~~ we were to maximize x_n, then the simplex method could indeed take 2^n iterations to terminate. Also, note that we get quite close to the unit cube by choosing $\epsilon > 0$ very small.

Interior point methods could take a much shorter path through the interior of the polytope.