Math 464: Linear Optimization (CAPS)

Credit Hours	3
Time	Tue+Thu 12:05–1:20 PM
Location	SPRK 333, VECS 125
Instructor	Bala Krishnamoorthy
Check-in Hours	Tue 3–4 PM, Wed 1:30–2:30 PM (Zoom)
Email	kbala@wsu.edu
Phone	360-546 9167
Course web page	https://www.wsu.edu/faculty/bkrishna/Math464.html
Text	Dimitris Bertsimas and John N. Tsitsiklis Introduction to Linear Optimization Athena Scientific. ISBN: 1-866529-19-1

Description of the Course

Linear optimization (or linear programming, LP) is the fundamental branch of optimization, with applications to many areas including life sciences, computer science, defense, finance, telecommunications, transportation, etc. Other types of optimization typically use LP as the underlying model. This course will provide an integrated view of the theory, solution techniques, and applications of linear optimization. There will be a fair bit of emphasis on theorems and their proofs. The treatment of most topics will begin with a geometric point of view, followed by the development of the solution techniques (algorithms), which are described using linear algebra. A background in linear algebra and multivariate calculus is assumed. Topics covered could include linear programming formulations, geometry of linear programming, the simplex method, duality, sensitivity analysis, integer programming, and basics of interior point methods. Apart from problems involving proofs, the student will use Matlab or another programming language (e.g., Python) for implementing some of the computations and algorithms. A state-of-the-art modeling software such as AMPL will also be introduced for solving problems modeling real life situations.

Prerequisites

Grades of **C** or better in Math 220: Introductory Linear Algebra, or equivalent, and in Math 273: Calculus of Functions of Several Variables, or equivalent, are required. While Math 301: Introduction to Mathematical Reasoning (Writing Proofs) is not required as an official pre-req, **there will be proof-type problems in this class**. Familiarity with a standard software program/package such as Matlab (or Python) would be quite useful. This course is open to students majoring in disciplines other than Mathematics.

Organization and Grading

There will be **around ten homework assignments**. Each assignment will be handed out at least one week before the day on which it will be due. Several problems will require the student to provide mathematical proofs and other supporting arguments for claims made on the theoretical and computational aspects of optimization. Some of the problems will also involve working with Octave (Matlab) or Python, or a similar programming language, and the modeling language AMPL.

Discussion of homework problems with others is allowed, but each person should hand in his or her own written solutions and programs.

There will be no exams, and hence doing well on your homework assignments is important in order to get a good grade in this course. Apart from the homework, there will be **one course project**. The project will involve the complete implementation of a solution methodology for linear optimization problems, e.g., the revised simplex method, in Matlab (or another programming language). Students should work on the project individually, and submit a formal written report along with associated software code.

- Homework: 65 %,
- project: 35 %.

The least homework grade **from among those turned in** will be dropped.

For all graded work, you will be evaluated on

$\Rightarrow$ how well you understood the underlying mathematical concepts,
$\Rightarrow$ whether you correctly applied the concepts to model the problem in hand, and
$\Rightarrow$ whether you provide correct and clear exposition and description of the above aspects, and of the results sought in the case of each problem.

Grades for the course will be assigned according to the following scale for the total scores obtained in the course. $\geq$92: A; 88–91.9: A$-$; 83–87.9: B+; 78–82.9: B; 73–77.9: B$-$; 68–72.9: C+; 63–67.9: C; 58–62.9: C$-$; 53–57.9: D+; 45–52.9: D; and $\leq$44.9: F.

Attendance Policy: I will not record your attendance. But, it is essential to not miss any of the lectures if you want to do well in this course. As such, you are encouraged to attend class regularly.

Software

There will be some homework problems involving the use of the computer package Matlab (or an alternative package such as Python). Some background in the use of Matlab or Python (or another similar package or language) will be helpful. You could learn the necessary basics as part of the assignments in this class (I am happy to help).

The modeling software AMPL will be introduced. You will be given access to an educational version of AMPL specific for this course (details will be provided soon). While this version will have no limitations, the license will expire at the end of the semester. You are welcome to download a free demo version from ampl.com, which has restriction on problem sizes (but no time limit on the license). Necessary documentation will be provided in the form of class handouts.

Academic Integrity: I encourage discussion of homework problems with others. But each student should submit their own (hand or type) written solutions and/or computer programs (codes). You might search the internet for finding materials to enhance your understanding. If you use such material to assist in your homework submission, you **should** cite the relevant sources. Plagiarism or cheating will **not** be tolerated. In particular, do not copy blindly from internet sources! Such behavior is easy to detect, and will result in a zero grade for the item in question and possibly a failing grade for the entire course.

Student Learning Outcomes and Assessment

A student completing this course will achieve the following learning goals.

- **Depth, breadth, and integration of learning:**
  - Develop a thorough understanding of relevant concepts from various fields of mathematics including **real analysis, geometry, multivariate calculus, linear algebra, combinatorics,** and **computational mathematics** that are employed in linear optimization.
  - Develop skills to model and solve problems from **other disciplinary areas** such as **business, economics, engineering, life sciences,** and **social sciences** as optimization problems.
  - Develop expertise in the use of **software tools** for **analyzing data from real life applications** that could be modeled using optimization methods.

- **Creative and Critical thinking:**
  - **Acquire** the necessary **competencies** for **solving problems** in the context of theory, computation, and applications of linear optimization. This goal would often require the **innovative use** of the relevant mathematical concepts for developing efficient models, theories, and algorithms.
  - Be able to **critically evaluate** the relative merits of various relevant approaches to solve problems in optimization, e.g., comparing the performances of the simplex method (Chapter 3) and interior point methods (Chapter 9).

- **Quantitative Reasoning:**
  - Acquire the skills to **develop optimization models**, e.g., linear or integer optimization problems, that **represent** the relevant information in an application. Become well versed in assessing the correctness of such models, and in interpreting the results from solving these models (**Calculation**).
  - Acquire the skills to **test** the relevant **assumptions** made for modeling the situations in question as optimization problems.

- **Information Literacy:**
  - Acquire skills to **seek, filter, and use information** from multiple exterior sources including **books, journals, and the internet** to assemble data as well as develop appropriate optimization models for various problems.
  - Become proficient in the **use of state-of-the-art software** for implementing and solving optimization problems, which are of sizes typically observed in real life (as opposed to just toy problems).

- **Communication skills (written and oral):**
  - Produce well structured **written reports** for the project, which describe the background, assumptions, methods, and results in a format typical of (short) papers in the field of optimization.
  - Provide sound mathematical arguments for the proof-type exercises in the homework assignments.
  - Participate in discussion during lecture in class.

Most of these goals will be assessed in an integrated fashion throughout the course. Relevant activities for the **assessment** of specific learning goals are tabulated below. **Evaluation** of each learning goal will be performed through **grading** the homework assignments, project reports, and the project presentations.

Learning goal	relevant activities
Depth, breadth, and integration of learning	all lectures, homework, projects
Information literacy	projects, homework problems using software
Applications to other disciplines	LP/IP formulations (Chap. 1,10), AMPL project
Use of software tools	MATLAB/AMPL homework problems, projects
Creative and critical thinking	all homework, projects
Quantitative reasoning	proof problems in homework, project
Communication skills (written)	project reports, proof problems in homework
Communication skills (oral)	in-class participation

Major Student Work Product

Students will be required to submit a detailed report for the project. This report will be used as the major student work product of this class. This report will test the student's achievement of *all* CAPS learning outcomes listed in the table above, except oral communication skills.

Tentative Schedule, and Topics Covered

We will cover sections from Chapters 1–4 and 9 from the book Introduction to Linear Optimization by Bertsimas and Tsitsiklis (BT-ILO). Sections from BT-ILO are indicated as, e.g., (§2.1–2).

Week	Lec #	Date	Details
1	1	Tue, Jan 10	intro, motivation from calculus, sample problem
	2	Thu, Jan 12	solution in AMPL, general and standard forms of LP (§1.1)
2	3	Tue, Jan 17	conversion to standard form, LP formulations (§1.2)
	4	Thu, Jan 19	LP formulations, convex and PL functions (§1.3) **[HW 1 Due]**
3	5	Tue, Jan 24	graphical solution, cases of LP (§1.4),
	6	Thu, Jan 26	more on cases of LP, linear algebra basics (§1.5) **[HW 2 Due]**
4	7	Tue, Jan 31	polyhedra, convex sets, basic feasible solutions (bfs) (§2.1–2)
	8	Thu, Feb 2	vertex, extreme point, bfs, and equivalence (§2.2) **[HW 3 Due]**
5	9	Tue, Feb 7	standard form of polyhedra (§2.3), degeneracy in LP (§2.4)
	10	Thu, Feb 9	existence of extreme points (§2.5) **[HW 4 Due]**
6	11	Tue, Feb 14	optimality of extreme points (§2.6)
	12	Thu, Feb 16	simplex method: optimality conditions (§3.1) **[HW 5 Due]**
7	13	Tue, Feb 21	simplex method (§3.2–3)
	14	Thu, Feb 23	simplex method (§3.3)
8	15	Tue, Feb 28	Bland's rule (§3.4), big-M method **[HW 6 Due]**
	16	Thu, Mar 2	finding initial bfs (§3.5)
9	17	Tue, Mar 7	tableau and revised simplex methods
	18	Thu, Mar 9	cycling in simplex method **[HW 7 Due]**
10		Tue, Mar 14	*Spring break*
		Thu, Mar 16	*Spring break*
11	19	Tue, Mar 21	more on simplex method (§3.*)
	20	Thu, Mar 23	more on simplex method (§3.*)
12	21	Tue, Mar 28	LP duality (§4.1-2) **[HW 8 Due]**
	22	Thu, Mar 30	LP duality, dual theorem (§4.3)
13	23	Tue, Apr 4	dual theorem, marginal costs (§4.3-4)
	24	Thu, Apr 6	dual simplex method (§4.5) **[HW 9 Due]**
14	25	Tue, Apr 11	Farkas' lemma (§4.6)
	26	Thu, Apr 13	Farkas' lemma (§4.6)
15	27	Tue, Apr 18	interior point methods, affine scaling (§9.*) **[HW 10 Due]**
	28	Thu, Apr 20	more on interior point methods (§9.*)
16	29	Tue, Apr 25	other topics
	30	Thu, Apr 27	other topics
17		Tue, May 2	**[Project due]**

WSU Systemwide Policies and Statements

Reasonable Accommodation

Reasonable accommodations are available for students with documented disabilities or chronic medical or psychological conditions. If you have such a condition and need accommodations to fully participate in this class, please visit your campus' Access Center/Services website to follow published procedures to request accommodations. Students may also contact their campus offices to schedule an appointment with an Access Advisor. All disability related accommodations are to be approved through the Access Center/Services on your campus. It is a university expectation that students connect with instructors (via email, Zoom, or in person) to discuss logistics within two weeks after they have officially requested their accommodations.

For more information, contact an Access Advisor on your home campus:

- Pullman, WSU Global Campus, Everett, Bremerton, and Puyallup: 509-335-3417 Access Center (https://www.accesscenter.wsu.edu) or email at access.center@wsu.edu
- Vancouver: 360-546-9739 Access Center (https://studentaffairs.vancouver.wsu.edu/access-center) or email van.access.center@wsu.edu.

Religious Accommodation

Washington State University reasonably accommodates absences allowing for students to take holidays for reasons of faith or conscience or organized activities conducted under the auspices of a religious denomination, church, or religious organization. Reasonable accommodation requires the student to coordinate with the instructor on scheduling examinations or other activities necessary for course completion. Students requesting accommodation must provide written notification within the first two weeks of the beginning of the course and include specific dates for absences. Approved accommodations for absences will not adversely impact student grades. Absence from classes or examinations for religious reasons does not relieve students from responsibility for any part of the course work required during the period of absence. Students who feel they have been treated unfairly in terms of this accommodation may refer to Academic Regulation 104 – Academic Complaint Procedures.

Academic Integrity

All members of the university community share responsibility for maintaining and promoting the principles of integrity in all activities, including academic integrity and honest scholarship. Students are responsible for understanding the full Academic Integrity Statement. Students who violate WSU's Academic Integrity Policy (identified in WAC 504-26-010(3) and -404) will receive a failing grade for the assignment or for the whole course, will not have the option to withdraw from the course pending an appeal, and will be reported to the Center for Community Standards. If you have any questions about what is and is not allowed in this course, ask your course instructor.

Safety and Emergency Notification

Classroom and campus safety are of paramount importance at Washington State University, and are the shared responsibility of the entire campus population. WSU urges students to follow the "Alert, Assess, Act," protocol for all types of emergencies and the "Run, Hide, Fight" response for an active shooter incident. Remain ALERT (through direct observation or emergency notification), ASSESS your specific situation, and ACT in the most appropriate way to assure your own safety (and the safety of others if you are able). Please sign up for emergency alerts on your account at MyWSU. For more information on this subject, campus safety, and related topics, please view the FBI's Run, Hide, Fight video and visit the WSU safety portal.

Full details can be found at https://provost.wsu.edu/classroom-safety/.

Severe Weather

For severe weather alerts, see http://alert.wsu.edu/ and https://oem.wsu.edu/emergency-procedures/severe-weather/. In the event of severe weather affecting university operations, guidance will be issued through the alert system.

WSU Vancouver Statement: In the event that an adverse weather event (e.g., snow or ice) or natural hazard that poses a safety risk occurs, you should take personal safety into account when deciding whether you can travel safely to and from campus, taking local conditions into account. If campus remains open and your instructor decides to cancel the face-to-face meeting and substitute an alternative learning activity, you will be notified by your instructor via email or through Blackboard within a reasonable time after the decision to open or close campus has been made. Instructions regarding any alternative learning options or assignments will be communicated in a timely manner. If travel to campus is not possible due to adverse regional conditions, allowances to course attendance policy and scheduled assignments, including exams and quizzes, will be made. Students who attempt to gain advantage through abuse of this policy (e.g., by providing an instructor with false information) may be referred to the Center for Community Standards for disciplinary action. If a student encounters an issue with an instructor, the student should first talk with the instructor. If the issue cannot be resolved, the student should follow the reporting violations of policies outlined on the student affairs website.

Discrimination and Harassment

Discrimination, including discriminatory harassment, sexual harassment, and sexual misconduct (including stalking, intimate partner violence, and sexual violence) is prohibited at WSU (See WSU Policy Prohibiting Discrimination and Harassment (Executive Policy 15) and WSU Standards of Conduct for Students). If you feel you have experienced or have witnessed discriminatory conduct, you can contact the WSU Compliance & Civil Rights (CCR) and/or the WSU Title IX Coordinator at 509-335-8288 to discuss resources, including confidential resources, and reporting options. (Visit ccr.wsu.edu for more information). Most WSU employees, including faculty, who have information regarding sexual harassment or sexual misconduct are required to report the information to CCR or a designated Title IX Coordinator or Liaison. (Visit ccr.wsu.edu/reporting-requirements for more info).

Lauren's Promise

I will listen and believe you if someone is threatening you. Lauren McCluskey, a 21-year-old honors student athlete, was murdered on Oct. 22, 2018, on the University of Utah campus by a man she briefly dated. *We must all take actions to ensure that this never happens again.*

If you are in immediate danger, call 911.

If you are experiencing sexual assault, domestic violence, stalking, discrimination or harassment, you have support and options. If you share information with me, please know that I am required to reach out to the Title IX Coordinator in WSU Compliance and Civil Rights (CCR), and CCR will reach out to you with information about on and off campus reporting options and resources. CCR is a system-wide resource (all campuses) which is available for intake consultations for you to learn more about available support. You can reach them directly at 509-335-8288, ccr@wsu.edu, or report online (anonymous reports accepted).

You can also speak to a victim advocate, a medical provider, or counselor confidentially about your concerns. Advocates help survivors of crime determine their own needs in regards to their physical and emotional health, reporting options, and academic concerns. At no cost, advocates connect survivors to campus and community services, and provide accompaniment to important appointments (court, hospital, and police) and support throughout the process. For a list of confidential victim advocates and medical providers, please visit CCR Resources.

WSU Police Department (WSU PD) officers and campus security will treat victims of sexual assault, domestic violence, stalking, hate crimes, and other crimes with respect and dignity. WSU PD, campus security departments, CCR, and victim advocates can also help you with safety planning.

Resources for Students

In Pullman

- Student Care Network: studentcare.wsu.edu
- Cougar Transit: 978 267-7233
- WSU Counseling and Psychological Services (CAPS): 509 335-2159
- Suicide Prevention Hotline: 800 273-8255
- Crisis Text Line: Text HOME to 741741
- WSU Pullman Police: 509 335-8548
- Pullman Police (Non-Emergency): 509 332-2521
- WSU Office of Civil Rights Compliance & Investigation: 509 335-8288
- Alternatives to Violence on the Palouse: 877 334-2887
- Pullman 24-Hour Crisis Line: 509 334-1133

In Vancouver

- Students may apply for grant assistance to cover technology or COVID-19 related educational costs through the Student Emergency Funding request form: https://studentaffairs.vancouver.wsu.edu/financial-aid
- The Laptop Loaner Program will continue in Spring 2022. To apply, students should visit: https://www.vancouver.wsu.edu/information-technology/new-student-tech-guide
- Tech Help for Students: http://wsuvtech4students.org/
- Access Campus Software Remotely with AppStream: https://www.vancouver.wsu.edu/information-technology/access-campus-software-remotely-appstream
- Cougar Food Pantry: We know that it can be hard for students to make ends meet when paying for college and living on a tight budget. The Cougar Food Pantry can help. The pantry provides free, nonperishable food items for WSU Vancouver students in need. The process is simple, anonymous and judgement-free. Learn more and request food at https://studentaffairs.vancouver.wsu.edu/cougar-food-pantry.