Algebraic Topology (Fall 2025): Homework 2

- You **must email your submission** as a **PDF file** to kbala@wsu.edu. You are welcome to write answers by hand, and scan or take photos of the writings. Put all the images on a PDF file, though.
- Your file name should identify you **in the following manner**. For instance, if you are Scott Tenorman, you should name your submission ScottTenorman_Hw2.pdf (and **NOT** "Scott Tenorman*pdf" or Scott**t**enorman_*pdf or ...). Please avoid white spaces in the file name :-).
- **Begin the SUBJECT of your email submission with the same `FirstnameLastname`, e.g., "ScottTenorman Hw2 submission".**
- The book of Munkres (Elements of Algebraic Topology) is denoted [M].
- The total points (given in parentheses) add up to 110.
- **This homework is due by 11:59 PM on Thursday, September 11**.

1. (20) Given a simplicial complex K, show that each point $\mathbf{x} \in |K|$, its underlying space, lies in the interior of exactly one simplex of K.
2. (25) **[M] Prob 4(a), Page 7:** Let σ be the simplex spanned by $\{\mathbf{a}_0, \dots, \mathbf{a}_n\}$, τ its face spanned by $\{\mathbf{a}_0, \dots, \mathbf{a}_p\}$ for $p < n$, and ρ its face spanned by $\{\mathbf{a}_{p+1}, \dots, \mathbf{a}_n\}$. We call ρ the face of σ *opposite* τ. Show that σ is the union of all line segments joining points of τ to points of ρ, and that two such line segments intersect in at most one common end point.
3. (20) **[M] Prob 5(b), Page 7:** Let U be a bounded open set in $\mathbb{R}^d$ that is **star-convex** relative to $\mathbf{0}$ (origin), i.e., for every $\mathbf{x} \in U$ the line segment from $\mathbf{0}$ to $\mathbf{x}$ lies in U. Show by example that $\overline{U}$ (closure of U) need not be homeomorphic to $\mathbb{B}^d$, the d-dimensional unit ball.
4. (25) Show that the vertices $\mathbf{a}_0, \dots, \mathbf{a}_n$ of a simplicial complex span a simplex of K if and only if the intersection of their stars is nonempty.
5. (20) **[M] Prob 2, Page 14:** Show that in general, the star and closed star are path connected, i.e., there exists a path between any two points in the set.