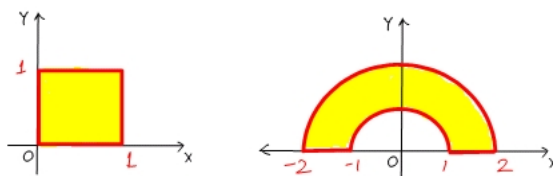


## Computational Topology (Spring 2026): Homework 1

- You **must email your submission** as a **PDF file** to kbala@wsu.edu. You are welcome to write answers by hand and scan the pages. Put all the images on a PDF file, though.
  - Your file name should identify you in the following manner. If you are Leopold Stotch, you should **name your submission LeopoldStotch\_Hw1.pdf**. If you want to add more bits to the title, e.g., Math529, you could name it LeopoldStotch\_Math529\_Hw1.pdf, for instance. But you should **start the file name with LeopoldStotch. And please avoid white spaces in the file name.**
  - Begin the SUBJECT of your email submission with the same FirstnameLastname, expression, e.g., "LeopoldStotch Hw1 submission".**
  - This homework is due by 10:00 PM on Tuesday, January 27.**
- (20) Meet with me briefly (in person or on Zoom). Check-in hours (for this class or Math 565) work best, but contact me if you want to meet at another time. You do **not** have to contact me if you plan to show up during one of the scheduled check-in hours.
  - (20) Show that the following two sets are homeomorphic by explicitly specifying a homeomorphism, i.e., a continuous function  $f$  from one set to the other, and its inverse.



- (25) Let us define a *neighborhood* of a point as any *open* set that contains the point. Consider a set  $A$  that is a subset of  $\mathbb{R}^2$ . We define a point  $x \in \mathbb{R}^2$  is **near** the set  $A$  if every neighborhood of  $x$  contains a point of  $A$ , i.e., intersects  $A$ . We denote this definition by  $x \leftarrow A$ .

Let  $A, B$  be sets in  $\mathbb{R}^2$ , and  $x \in \mathbb{R}^2$ . Prove that if  $x \leftarrow A$  or  $x \leftarrow B$ , then  $x \leftarrow A \cup B$ . Also prove the converse, i.e., that if  $x \leftarrow A \cup B$ , then either  $x \leftarrow A$  or  $x \leftarrow B$ , or both.

- (20) State if each of the following objects is a manifold, a manifold with boundary, or neither. If it is one of the first two cases, specify the dimension of the manifold.

