Computational Topology (Spring 2026): Homework 3

- You **must email your submission** as a **PDF file** to kbala@wsu.edu. You are welcome to write answers by hand, and scan or take photos of the writings. Put all the images on a PDF file, though.
- Your file name should identify you in this manner. If you are Billy Circlovich, you should name your submission BillyCirclovich_Hw3.pdf. **Please start your name in this format. If you want to add details to the title, you could name it BillyCirclovich_Math529_Hw3.pdf, for instance. Please avoid white spaces in the file name :-).**
- If you want to **send additional files**, e.g., containing your code, then you should **include all files in a compressed folder**, e.g., zipped or tar-gzipped, named in the same manner, i.e., **BillyCirclovich_Hw3.zip**. You could include the PDF file inside this zipped folder, along with the other files.
- WSU Email will **not allow attachments of .py files**, even inside Zipped folders! You can save such files as .txt files, or include corresponding Jupyter notebooks (.ipynb files) instead.
- **Begin the SUBJECT of your email submission with the same `FirstnameLastname`, expression, e.g., "BillyCirclovich Hw3 submission".**
- **This homework is due by 10:00 PM on Tuesday, February 24**.

1. (30) A *flag* in a simplicial complex K in $\mathbb{R}^d$ is a nested sequence of proper faces $\sigma_0 \prec \sigma_1 \prec \cdots \prec \sigma_k$. The collection of flags in K forms an abstract simplicial complex A, called the *order complex* of K. Prove that the order complex A of K has a geometric realization in $\mathbb{R}^d$.

Use the specified datasets for the subsequent problems. The set of triangles in a 2-complex is given in the file Triangles.txt. The $\{x, y, z\}$ coordinates of the vertices used in the triangles are given in the file Vertices.txt. The vertex labels as used in the list of triangles correspond to the line numbers in the file Vertices.txt. For instance, the coordinates of the three vertices in the triangle $[v_3,\ v_{256},\ v_{145}]$ (specified in a single line as $[3,\ 256,\ 145]$) are listed in lines 3, 256, and 145, respectively, in the vertex coordinates file.

2. (20) Find the Euler characteristic of the given 2-complex. Explain how you are finding the (list of) vertices and edges (so as to count the numbers of them).

3. (30) Is the 2-complex a surface? If not, could you make it a surface by adding a few missing triangles?

4. (50) Assuming you have a surface from the previous step, can you orient the surface? You could try to propagate a chosen orientation from a single triangle to all other triangles. What standard surface, if any, is this one homeomorphic to? Explain the steps/procedure you use to propagate orientations.

5. (30) Produce a few views of the patched up surface using TetView, or another meshing software (or in Matlab or Python, for instance). Include these views as images in your report.