

## Computational Topology (Spring 2026): Homework 6

- You **must email your submission** as a **PDF file** to kbala@wsu.edu. You are welcome to write answers by hand, and scan or take photos of the writings. Put all the images on a PDF file, though.
- Your main file's name should identify you in this manner. If you are Eric Theodore Cartman, you should name your submission EricCartman\_Hw6.pdf. **Please start your name in this format. If you want to add details to the title, you could name it EricCartman\_Math529\_Hw6.pdf, for instance. Please avoid white spaces in the file name :-).**
- **Begin the SUBJECT of your email submission with the same FirstnameLastname, expression, e.g., "EricCartman Hw6 submission".**
- **This homework is due by 9:59 PM on Monday, April 20.**

1. (30) Let  $K$  be the simplicial complex made of a tetrahedron and its faces. Apply the matrix reduction algorithm to the filtration of  $K$  that adds simplices in the order of their dimensions. For the three diagrams you get (in the three relevant dimensions), do any of them depend on the way you order simplices of the same dimension?
2. (35) The main step in the matrix reduction implementation of the persistence algorithm (discussed in Lecture 23 and Lecture 24) was the following.

$$\begin{array}{l} \text{while } \exists j' < j \text{ with } \text{low}(j') = \text{low}(j) \text{ do} \\ \quad \text{add column } j' \text{ to column } j \end{array}$$

Consider a variant of the matrix reduction implementation of the persistence algorithm where you add columns **to the right** of column  $j$  instead of from the left. The condition will be

$$\begin{array}{l} \text{while } \exists j_0 > j \text{ with } \text{low}(j_0) = \text{low}(j) \text{ do} \\ \quad \text{add column } j \text{ to column } j_0 \end{array}$$

- (a) Show that this implementation generates the same lowest 1's as the original implementation.
- (b) Give an example complex where the two implementations give different reduced matrices  $R$ .