Network Optimization (Fall 2024): Homework 6 Part 2

- AMO stands for the text (Ahuja, Magnanti, Orlin). Exercises and page numbers are listed from AMO.
- The total points (given in parentheses) add up to 80, which add to the 70 points from Homework 6.1. Math 566 students will be graded for 145 points, and Math 466 students for 125 points.
- **Send your submission to kbala@wsu.edu. Your main submission should be in a PDF file**. You are welcome to write answers by hand, and scan the writings (or take pictures of your writings) **into a PDF file**. You can also include output from running your program (for Problem 6) in the PDF file.
- Include all files—the main PDF and any program (.m) files—in a **zipped** folder.
- **You will not be allowed to include .py files in your email attachment. Notebooks (.ipynb files) are allowed. Another option is to rename your .py files as .txt instead.**
- **Your folder name should identify you in the following manner. If you are Wendy Testaberger, you should name your folder WendyTestaberger_Hw6_2.zip. If you want to add more bits to the title, e.g., Math566, you could name it WendyTestaberger_Math566_Hw6_2.zip, for instance. But you should start the file name with WendyTestaberger. Please avoid white spaces in the file name.**
- **Begin the SUBJECT of your email submission with the same `FirstnameLastname`, expression, e.g., "WendyTestaberger Hw6_2 submission".**
- **This homework is due by 11:59 PM on Thursday, October 10**.

4. (25) **[G]** AMO 4.44 (pages 130–131).
   See Page 2 of this Homework PDF for a clarification.

5. (25) AMO 5.7 (page 158–159).
   TYPO!: Bellman's equations should read

$$d(j) = \min\{d(i) + c_{ij} \ : \ (i,j) \in \mathbf{AI}(j)\} \ \text{ for all } j \in N.$$

6. (30) Write a Matlab (or another language/package) function that takes as input the forward star representation of a network and a starting node s, and finds the shortest path to all nodes using Dijkstra's algorithm. The program should output the `pred` vector as well as the shortest path lengths (the vector d). Name your file `Dijkstra_FirstnameLastname.m` (e.g., `Dijkstra_WendyTestaberger.m`) and include the file in your homework submission.

   - Demonstrate your code on the shortest path instance given in Figure 4.15 (a), Page 127. You **must** include the output for this instance in your submission. You could include the output in the main PDF file itself, or as part of the code submission file (as part of comments, for instance). But if you are sending in a separate file for the output, it must be **in text format** and should be named **`Output_Dijkstra_FirstnameLastname.txt`**
   - Your function must take **the entire Forward Star matrix as a single input**—from a text file, or it could be typed in as part of separate code that you turn in, for instance. In particular, **your code must not prompt the user to input each arc one at a time!**

Clarification on for Problem 4 (AMO Problem 4.44, pages 130–131)

The definition of *tolerances* as specified in AMO (as part of this problem statement) is not clear without making some modifications as well as assumptions. Consider the following cases.

If $(i, j) \in T$, both α_{ij} and β_{ij} could be finite or infinite depending on the structure of G. For instance, consider the simple network with one arc $(1, 2)$ connecting nodes 1 and 2. Indeed, the unique SP tree consists of $(1, 2)$ irrespective of the value of c_{12}. Hence, both α_{ij} and β_{ij} are infinite.

On the other hand, consider the network G with $N = \{1, 2, 3\}$, $A = \{(1, 2), (2, 3), (1, 3)\}$ with $c_{12} = 2$, $c_{23} = 3$, $c_{13} = 4$, and $s = 1$. The SP tree T consists of $(1, 2)$ and $(1, 3)$. But it will change to T' consisting of $(1, 2)$ and $(2, 3)$ if c_{12} is decreased to any value below 1. Thus $\beta_{12} = 1$, which is finite. Further, since $(1, 2)$ is the only path (arc) from $s = 1$ to node 2, $\alpha_{12} = \infty$ here!

On the other hand, we can see that for any value of decrease in c_{12} (equivalently, for any $\beta_{12} > 0$), the arc $(1, 2)$ would still be part of *some* SP tree of G. Hence we should use the following modification of the definition of tolerances for this problem.

The original definition (in AMO) does make sense for an arc $(i, j) \notin T$. But for $(i, j) \in T$, it makes more sense to consider the tolerances on c_{ij} for which that arc is still part of *some* SP tree of G, rather than for the original SP tree T to remain unchanged. Further, we will assume that there is an alternative s–j path that does not use (i, j).