

Math 466/566 (Fall 2026): Flow Decomposition Algorithm

INPUT : $G = (N, A)$ with arc flow $\mathbf{x}$ (m -vector of flows x_{ij}).

OUTPUT: $\mathcal{P}$ with $f(P) \forall P \in \mathcal{P}$, $\mathcal{W}$ with $f(W) \forall W \in \mathcal{W}$.

(here, $\mathcal{P}$ and $\mathcal{W}$ are the sets of paths and cycles, and f is flow).

Notation:

$\mathbf{y}$: intermediate flow

$G(\mathbf{y}) = (N(\mathbf{y}), A(\mathbf{y}))$: graph corresponding to flow $\mathbf{y}$

$A(\mathbf{y}) = \{(i, j) \in A \mid y_{ij} > 0\}$ (arcs with positive flow in $\mathbf{y}$)

$N(\mathbf{y}) = \{i \mid (i, j) \in A(\mathbf{y}) \text{ or } (j, i) \in A(\mathbf{y})\}$ (nodes incident to arcs in $A(\mathbf{y})$)

$\mathcal{S} = \{i \mid b(i) > 0\}$ (supply nodes)

$\mathcal{D} = \{i \mid b(i) < 0\}$ (demand nodes)

for $P \in \mathcal{P}$, $\Delta(P) = \min\{b(s), -b(t), \min\{y_{ij} \mid (i, j) \in P\}\}$ (capacity of path)

for $W \in \mathcal{W}$, $\Delta(W) = \min\{y_{ij} \mid (i, j) \in W\}$ (capacity of cycle)

Algorithm

$\mathbf{y} := \mathbf{x}$, $\mathcal{P} = \emptyset$, $\mathcal{W} = \emptyset$, assign $A(\mathbf{y})$, $N(\mathbf{y})$, $\mathcal{S}$, and $\mathcal{D}$. (Initialization)

while $\mathbf{y} \neq \mathbf{0}$ **do**

begin

$s = \text{Select}(\mathbf{y})$

$\text{Search}(s, \mathbf{y})$

if cycle W found **then do**

begin

$\mathcal{W} = \mathcal{W} \cup \{W\}$

$f(W) = \Delta(W)$

$y_{ij} = y_{ij} - \Delta(W) \forall (i, j) \in W$

 update $A(\mathbf{y})$, $N(\mathbf{y})$

end

if path P found **then do**

begin

$\mathcal{P} = \mathcal{P} \cup \{P\}$

$f(P) = \Delta(P)$

$y_{ij} = y_{ij} - \Delta(P) \forall (i, j) \in P$

$b(s) = b(s) - \Delta(P)$ (s is starting node of P)

$b(t) = b(t) + \Delta(P)$ (t is ending node of P)

 update $A(\mathbf{y})$, $N(\mathbf{y})$, $\mathcal{S}$, $\mathcal{D}$

end

end

Select($\mathbf{y}$)

if $\mathcal{S} \neq \emptyset$ **then** choose $s \in \mathcal{S}$;

else choose $s \in N(\mathbf{y})$;

Search($s, \mathbf{y}$)

 Do DFS starting with node s until finding a cycle W in $G(\mathbf{y})$

 or a path P in $G(\mathbf{y})$ from node s to a node $t \in \mathcal{D}$