

Integer Optimization (Spring 2025): Homework 1

- You **must email your submission** as a **PDF file** to kbala@wsu.edu. You are welcome to write answers by hand, and scan the writings.
 - Your **file name** should identify you in the following manner. If you are **Uncle Jimbo**, you should name your submission **UncleJimbo_Hw1.pdf**. If you want to add more bits to the title, e.g., Math567, you could name it UncleJimbo_Math567_Hw1.pdf, for instance. But you should **start the file name with UncleJimbo**. And **NOT “Uncle Jimbo” or “Uncle_Jimbo” or ...**
 - Begin the **SUBJECT of your email submission** with the same **FirstnameLastname**, e.g., **“UncleJimbo Hw1 submission”**.
 - The total points (given in parentheses) add up to 145. You will be graded for 140 points.
 - **This homework is due by 11:59 PM on Thursday, January 23.**
0. (15) Meet with me briefly (on Zoom or in person). Do so even if you have taken a class with me in the past.
1. (20) Let f_1, \dots, f_m be piecewise linear and convex functions from \mathbb{R}^n to \mathbb{R} . Define $f(\mathbf{x}) = \sum_{i=1}^m f_i(\mathbf{x})$, $\forall \mathbf{x} \in \mathbb{R}^n$. Show that f is also a **piecewise linear and convex** function.
2. (25) Reformulate the following problem as a linear optimization problem (LP).
- $$\begin{array}{ll} \min & 2x_1 + 3|x_2 - 10| \\ \text{s.t.} & |x_1 + 2| + |x_2| \leq 5. \end{array}$$
3. (30) You are interested in choosing from a set of seven Investments $\{1, \dots, 7\}$. Model each of the following conditions using 0-1 variables.
- (a) You cannot invest in all of them.
 - (b) You must choose at least one of them.
 - (c) Investment 2 cannot be chosen if Investment 3 is chosen.
 - (d) Investment 4 can be chosen only if Investment 1 is chosen.
 - (e) You must choose either both Investments 1 and 2, or neither.
 - (f) You must choose either at least one of Investments 1,2,3, or at least two of Investments 2,3,5,6.
4. (25) A set of n jobs must be carried out on a single machine, which can do only one job at a time. Job j takes p_j hours to complete and has weight w_j , for $j = 1, \dots, n$. Formulate as a mixed integer program (MIP) the scheduling problem to find the order in which to carry out the n jobs, so as to minimize the weighted sum of their starting times.
5. (30) Give an IP or MIP formulation to solve a 3×3 Sudoku puzzle, i.e., the puzzle on a 9×9 grid. Assume that a subset of the squares $S = \{(i, j)\}$ are already filled in with values f_{ij} . The rules of Sudoku specify that each of the 3×3 blocks (there are 9 of them arranged in a 3×3 grid) must contain each of the numbers $1, 2, \dots, 9$ exactly once. Similarly, each row and each column must contain the numbers $1, 2, \dots, 9$ exactly once.