Integer Optimization (Spring 2025): Homework 2

- You **must email your submission** as a **PDF file** to kbala@wsu.edu. You are welcome to write answers by hand, and scan the writings.
- Your **file name** should identify you in the following manner. If you are Stan Jarsh, you should name your submission StanJarsh_Hw2.pdf. If you want to add more bits to the title, e.g., Math567, you could name it StanJarsh_Math567_Hw2.pdf, for instance. But you should **start the file name with StanJarsh.** And it is **NOT "Stan Jarsh" or "Stan_Jarsh" or "StanleyJarsh" or ...**
- Begin the **SUBJECT of your email submission** with the same **`FirstnameLastname`**, e.g., **"StanJarsh Hw2 submission"**.
- The total points (given in parentheses) add up to 130. You will be graded for 125 points.
- **This homework is due by 11:59 PM on Friday, February 7**.

1. (20) Consider the uncapacitated lot sizing (ULS) problem discussed in class. Explain how to model the following modifications to the original problem.

   (a) We allow demand to be *backlogged*, i.e., some demand from period t could be satisfied at a future time period. One unit of demand from period t backlogged (or carried) over to the next period incurs a cost of b_t.

   (b) If production happens in two consecutive time periods, the number of units produced in the second time period should be at least as big as the number produced in the first of the two time periods.

2. (25) Give a formulation for the **interactive fixed charge function** using appropriate 0–1 variables (in addition to the original variables x_1, x_2). Similar to the basic fixed charge problem discussed in class, we now have $f(x_1, x_2)$ with

$$f(x_1,x_2) = \begin{cases} 0 \;, & \text{if } x_1 = 0, x_2 = 0, \\ f_1 \;, & \text{if } x_1 > 0, x_2 = 0, \\ f_2 \;, & \text{if } x_1 = 0, x_2 > 0, \text{ and} \\ f_{12}, & \text{if } x_1 > 0, x_2 > 0. \end{cases}$$

   Here, f_1, f_2, f_{12} are positive scalars and we assume

$$0 \le x_1 \le M_1, \quad 0 \le x_2 \le M_2, \text{ and} \tag{1}$$

$$f_1 \le f_{12}, \quad f_2 \le f_{12}. \tag{2}$$

   Both $f_{12} > f_1 + f_2$ and $f_{12} < f_1 + f_2$ can happen. Show that condition (2) is necessary for the formulation to be correct.

3. (25) Consider a piecewise linear (PL) function $f(x)$ defined on $[v_0, v_3]$ such that

$$f(v_0) = f_0, \; f(v_1) = f_{1,\ell}, \; f(v_2) = f_2, \text{ and } f(v_3) = f_3,$$

   as discussed in class, but there is a jump at v_1. So, $f(x)$ is linear with slope

$$s_2 = \frac{f_2 - f_{1,r}}{v_2 - v_1}$$

   for all $v_1 < x \le v_2$, where $f_{1,r} > f_{1,\ell}$. Give an MIP representation of $f(x)$, assuming it appears in a minimization objective. Prove that your formulation will not work if it appeared in a maximization objective.

4. (30) Model the following logical statements using their CNFs.

   (a) $(L_1 \wedge L_4) \vee (L_1 \wedge L_5) \vee (L_2 \wedge L_4) \vee (L_2 \wedge L_5) \vee (L_3 \wedge L_4) \vee (L_3 \wedge L_5) \vee \neg(L_2 \wedge L_5)$. What if the last negated substatement were $\neg(L_4 \wedge L_5)$ instead of $\neg(L_2 \wedge L_5)$?

   (b) $L_1 \vee \cdots \vee L_m \Leftrightarrow J_1 \wedge \cdots \wedge J_n$, where L_i means $x_i = 1$, J_k means $y_k = 1$, for 0–1 variables x_i and y_k.

5. (30) When modeling arbitrary disjunctions, prove that if **Assumption 2** (all the polyhedra have the same recession cone) is satisfied, then so is **Assumption 1** (there exist upper bound vectors $\mathbf{u}^i$). What about the other way around—does **Assumption 1** imply **Assumption 2**? See Lecture 4 scribe for the statements of the two assumptions.